Worksheet: Equilibrium of a Rigid Body under Coplanar Couples

Q1: If the couples $\mathbf{M}_1$ and $\mathbf{M}_2$ are in equilibrium, where $\mathbf{M}_1 = 50\mathbf{k}$, then find the value of $\mathbf{M}_1 - \mathbf{M}_2 = \_\_$.

Question Video

A $50\mathbf{k}$

B $-50\mathbf{k}$

C $100\mathbf{k}$

D 0

Q2: The moments, M_1 and M_2, of two couples satisfy the equation $M_1 + M_2 = 0$. Which of the following is therefore true?

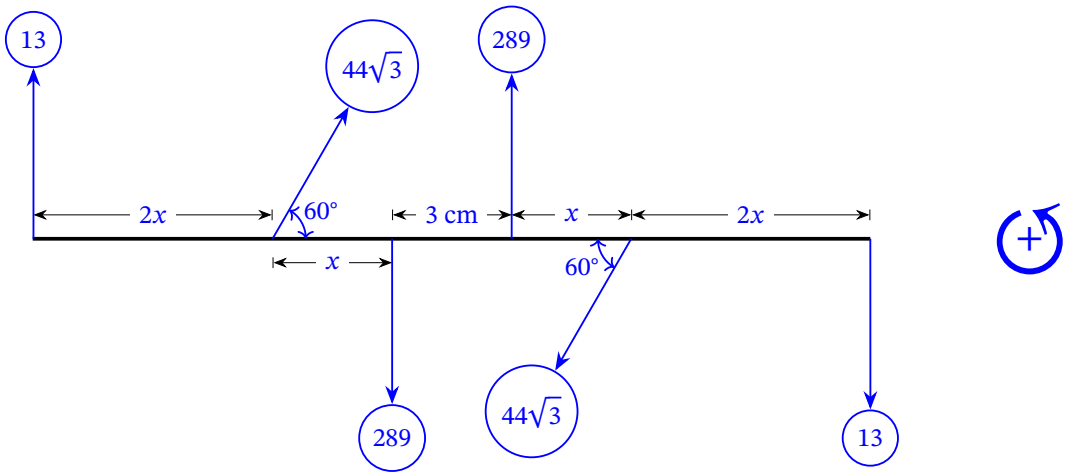
A the two couples are in equilibrium

B the two couples are equivalent to a force

C the two couples are equivalent

D the two couples are not in equilibrium

Q3: In the shown figure, forces of magnitudes 13, 13, $44\sqrt{3}$, $44\sqrt{3}$, 289, and 289 newtons are acting on a rod. Given that the rod is in equilibrium, and x is measured in centimetres, find the length of the rod.



A 30 cm

B 15 cm

C 21 cm

D 3 cm

Q4: The forces $\mathbf{F}_1 = 2\mathbf{i} + 7\mathbf{j}$, $\mathbf{F}_2 = a\mathbf{i} - 6\mathbf{j}$, and $\mathbf{F}_3 = 6\mathbf{i} + (b + 8)\mathbf{j}$ act on a particle, where $\mathbf{i}$ and $\mathbf{j}$ are two perpendicular unit vectors. Given that the system is in equilibrium, determine the values of a and b.

Question Video

A $a = 4, b = -9$

B $a = -8, b = -9$

C $a = -4, b = 7$

D $a = -8, b = 7$

E $a = -8, b = -1$

Q5: Three forces are acting on a particle. Two of them are given by $(2\mathbf{i} + \mathbf{j} + 3\mathbf{k})$ newtons and $(\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ newtons. Given that the particle remains at rest, find the third force.

A $(3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$ newtons

B $(-2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$ newtons

C $(\mathbf{i} + 4\mathbf{j} + \mathbf{k})$ newtons

D $(-\mathbf{i} - 4\mathbf{j} - \mathbf{k})$ newtons

E $(-3\mathbf{i} + 2\mathbf{j} - 5\mathbf{k})$ newtons

Q6: Given that $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are three coplanar forces in equilibrium meeting at a point, where $\mathbf{F}_1 = 5\mathbf{i} - 3\mathbf{j}$ and $\mathbf{F}_2 = 4\mathbf{i} - 14\mathbf{j}$, find $\mathbf{F}_3$.

Question Video

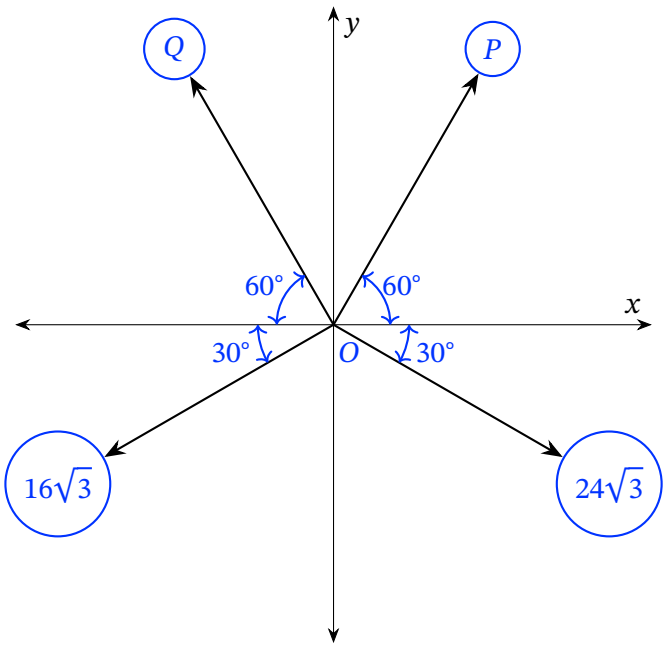
A $9\mathbf{i} - 17\mathbf{j}$

B $-9\mathbf{i} + 17\mathbf{j}$

C $-\mathbf{i} - 11\mathbf{j}$

D $\mathbf{i} + 11\mathbf{j}$

Q7: Forces of magnitudes P N, Q N, $16\sqrt{3}$ N, and $24\sqrt{3}$ N act at the point O as shown in the diagram. Given that the forces are in equilibrium, determine the values of P and Q.



A $P = 8, Q = 32$

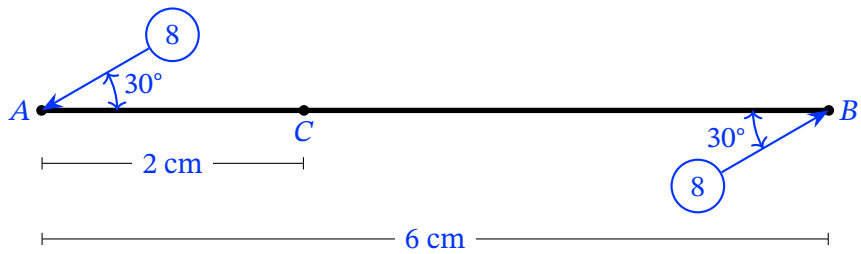
B $P = 16, Q = 24$

C $P = 8, Q = 48$

D $P = 24, Q = 16$

E $P = 32, Q = 8$

Q8: AB is a uniform rod with length 6 cm. It is free to rotate about a smooth nail in a small hole in the rod at a point C between A and B, where $AC = 2$ cm. The rod is in equilibrium, laying horizontally, under the action of two forces, each of magnitude 8 N, acting at either end at an angle of 30° with the rod as shown in the figure below. Find the weight of the rod W and the magnitude of the reaction of the nail R.



A $W = 24$ N, $R = 24$ N

B $W = 48\sqrt{3}$ N, $R = 48\sqrt{3}$ N

C $W = 48$ N, $R = 48$ N

D $W = 24\sqrt{3}$ N, $R = 24\sqrt{3}$ N

Q9: $ABCD$ is a rectangle, in which $AB = 27$ cm, and $BC = 18$ cm. Forces of magnitudes F_1, 14, F_2, and 14 newtons are acting along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, and $\overrightarrow{DA}$, respectively. If this system of forces is in equilibrium, determine the values of F_1 and F_2, such that the positive direction is $DCBA$.

A $F_1 = 14$ N, $F_2 = 14$ N

B $F_1 = 7$ N, $F_2 = 14$ N

C $F_1 = 9.33$ N, $F_2 = 21$ N

D $F_1 = 21$ N, $F_2 = 21$ N

Q10: A rod AB having a length of 72 cm is of negligible weight. C and D are two points on the rod that are 42 cm and 60 cm away from the end A respectively. Forces of magnitudes 380, F, 380, and F newtons are acting perpendicularly to the rod at the points A, C, D, and B respectively. Given that the two forces at A and B are acting on the rod in an opposite direction to those at C and D, and the rod is in equilibrium, determine the magnitude of the force that's denoted by F.

A 532 N

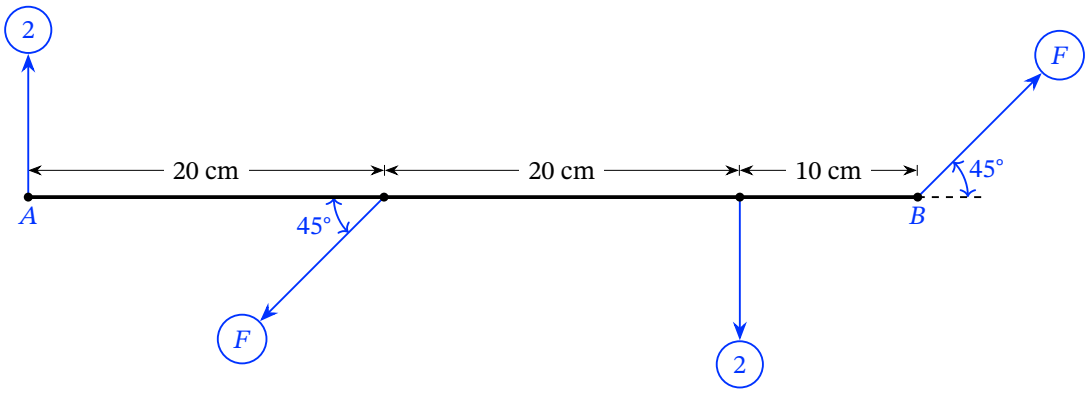
B 380 N

C 760 N

D 228 N

E 266 N

Q11: AB is a rod having a length of 50 cm and a negligible weight. Two coplanar pairs of forces are acting on the rod as shown in the figure. The first couple consists of two forces acting perpendicularly to the rod, each of magnitude 2 kg-wt, and the second couple consists of two forces, each of magnitude F. Determine the value of F that makes the rod in equilibrium.



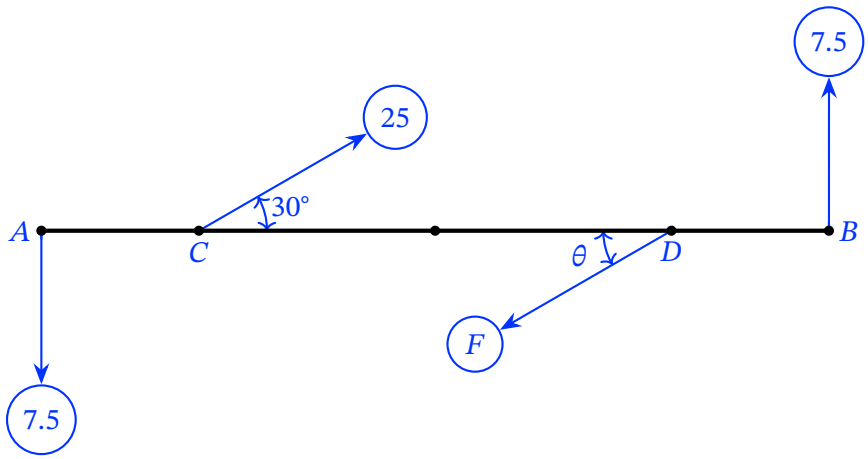
A $\frac{16\sqrt{2}}{3}$ kg-wt

B $\frac{8\sqrt{2}}{3}$ kg-wt

C $\frac{4\sqrt{2}}{3}$ kg-wt

D $\frac{10\sqrt{2}}{3}$ kg-wt

Q12: AB is a rod having a length of 90 cm and a negligible weight. It is suspended horizontally by a pin at its midpoint. Two forces, each is of a magnitude 7.5 N, are acting at its ends as shown in the figure. It is also pulled by a string, whose tension is 25 N, in a direction making an angle of 30° with the rod from point C. If a force F is acting on the rod at point D so that the rod is in a horizontal equilibrium position, find the magnitude of F, its direction θ, and the length of CD.



A $F = 25$ N, $\theta = 60°$, $CD = 27$ cm

B $F = 7.5$ N, $\theta = 30°$, $CD = 18\sqrt{3}$ cm

C $F = 25$ N, $\theta = 30°$, $CD = 18\sqrt{3}$ cm

D $F = 25$ N, $\theta = 60°$, $CD = 54$ cm

E $F = 25$ N, $\theta = 30°$, $CD = 54$ cm

Q13: AB is a rod of negligible weight, and length 54 cm. It is suspended horizontally by a pin at its midpoint. Forces of magnitude $68\sqrt{3}$ N act on each end, one of them vertically upwards at A and the other vertically downwards at B. The rod is pulled by a string, attached to it at point C, inclined at an angle of 60° to AB. The tension in the string has a magnitude of 192 N. The rod is kept in horizontal equilibrium by a fourth force F acting on the rod at point D with an angle of 60° to BA. Find the magnitude of F and the length of $\overline{DC}$.

A $F = 192$ N, $DC = 38.25$ cm

B $F = 192$ N, $DC = 19.12$ cm

C $F = 384$ N, $DC = 38.25$ cm

D $F = 384$ N, $DC = 19.12$ cm

Q14: The forces $(-7\mathbf{i} + 13\mathbf{j})$ N, $(a\mathbf{i} + \mathbf{j})$ N, and $(-5\mathbf{i} + (b-2)\mathbf{j})$ N are acting on a particle. Given that they are in equilibrium, what are the values of a and b?

A $a = 12, b = -16$

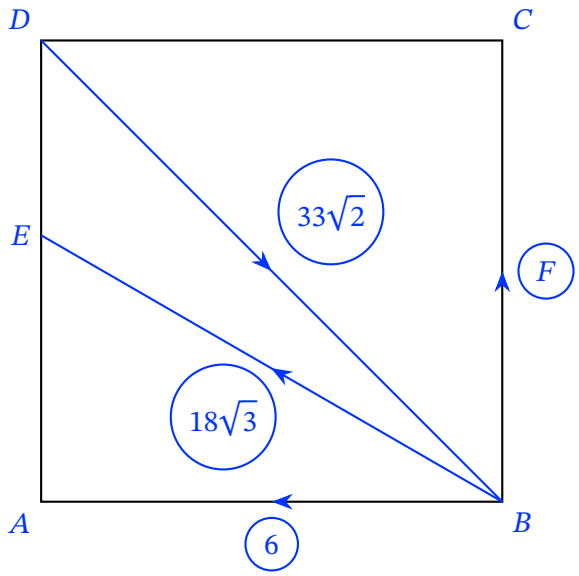
B $a = 2, b = -12$

C $a = -2, b = -16$

D $a = 12, b = -12$

E $a = 12, b = -14$

Q15: The diagram shows a square, $ABCD$, where E is a point on $\overline{AD}$. Forces of magnitudes 6 N, $18\sqrt{3}$ N, $33\sqrt{2}$ N and F N act at B as indicated on the diagram. Given that the forces are in equilibrium, find $m\angle ABE$ and the value of F to two decimal places.



A $m\angle ABE = 37°, F = 1.82$ N

B $m\angle ABE = 30°, F = 17.41$ N

C $m\angle ABE = 39°, F = 64.18$ N

D $m\angle ABE = 37°, F = 48.59$ N

Q16: Coplanar forces of magnitude 18 N, 5 N, F N, 9 N, K N, and 13 N are acting towards a particle, where the angle between each two consecutive forces is 60°. Find the magnitudes of F and K for the system to be in equilibrium.

A $F = 22$ N, $K = 14$ N

B $F = 17$ N, $K = 4$ N

C $F = 27$ N, $K = 14$ N

D $F = 47.5$ N, $K = 50$ N

Q17: A body weighing 61 kg-wt is placed on a smooth plane inclined at 30° to the horizontal. Given that it is maintained in a state of equilibrium by means of a force inclined at 60° above the horizontal, find the magnitude of the force F and the reaction R of the plane.

A $F = \frac{61\sqrt{6}}{6}$ kg-wt, $R = \frac{61\sqrt{6}}{6}$ kg-wt

B $F = \frac{61\sqrt{3}}{3}$ kg-wt, $R = \frac{61\sqrt{3}}{3}$ kg-wt

C $F = \frac{61\sqrt{6}}{6}$ kg-wt, $R = \frac{61\sqrt{3}}{3}$ kg-wt

D $F = \frac{61\sqrt{3}}{3}$ kg-wt, $R = \frac{122}{3}$ kg-wt