

Worksheet: Combining the Product, Quotient, and Chain Rules



Q1: Find the first derivative of the function $y = 9x + 5x \left(4x^2 + \frac{5}{x}\right)^2$.

A $80x^4 + 200x + 125x^{-2} + 9$

B $400x^4 + 400x - 125x^{-2} + 9$

C $400x^4 + 400x + 125x^{-2} + 9$

D $320x^4 + 200x - 250x^{-2} + 9$



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Q2: Differentiate $y = \sqrt[3]{x}(-2x + 1)$.

A $y' = -\frac{8\sqrt[3]{x}}{3} + \frac{1}{3x^{\frac{2}{3}}}$

B $y' = -2x^{\frac{7}{3}} + x^{\frac{4}{3}}$

C $y' = -\frac{2\sqrt[3]{x}}{3} - \frac{2}{3x^{\frac{2}{3}}}$

D $y' = -\frac{8x^{\frac{7}{3}}}{3} + \frac{x^{\frac{4}{3}}}{3}$

E $y' = -2\sqrt[3]{x} + \frac{1}{x^{\frac{2}{3}}}$

Q3: Find the first derivative of $y = \sqrt{\frac{\sec 9x - 4}{\sec 9x + 4}}$.

- A $\frac{36 \sec^2 9x \tan^2 9x}{(\sec 9x - 4)(\sec 9x + 4)} \sqrt{\frac{\sec 9x - 4}{\sec 9x + 4}}$
- B $\frac{36 \tan^2 9x}{(\sec 9x - 4)(\sec 9x + 4)} \sqrt{\frac{\sec 9x - 4}{\sec 9x + 4}}$
- C $-\frac{36 \sec 9x \tan 9x}{(\sec 9x - 4)(\sec 9x + 4)} \sqrt{\frac{\sec 9x - 4}{\sec 9x + 4}}$
- D $\frac{36 \sec 9x \tan 9x}{(\sec 9x - 4)(\sec 9x + 4)} \sqrt{\frac{\sec 9x - 4}{\sec 9x + 4}}$

Q4: Determine the derivative of the function $s(t) = \sqrt{\frac{-\sin t + 7}{-\cos t + 7}}$.

- A $s'(t) = -\frac{7 \sin t + 7 \cos t + 1}{2\sqrt{-\sin t + 7}(-\cos t + 7)^{\frac{3}{2}}}$
- B $s'(t) = -\frac{7 \sin t + 7 \cos t + 1}{2\sqrt{-\sin t + 7}(-\cos t + 7)^{\frac{2}{3}}}$
- C $s'(t) = -\frac{7 \sin t + 7 \cos t - 1}{2\sqrt{-\sin t + 7}(-\cos t + 7)^{\frac{2}{3}}}$
- D $s'(t) = \frac{7 \sin t + 7 \cos t - 1}{2\sqrt{-\sin t + 7}(-\cos t + 7)^{\frac{3}{2}}}$
- E $s'(t) = -\frac{7 \sin t + 7 \cos t - 1}{2\sqrt{-\sin t + 7}(-\cos t + 7)^{\frac{3}{2}}}$

Q5: Let $g(x) = h[6x - f(x)]$. If $f(-4) = -5$, $f'(-4) = 2$, and $h'(-19) = -2$, find $g'(-4)$.

A 2

B -8

C -4

D 10

E -10

Q6: Find $\frac{dy}{dx}$ at $x = 2$ when $y = (x^2 + x - 2)^2 (-3x^2 + 7x - 1)^5$.

A 60

B 15

C -360

D -90

Q7: Find the first derivative of $y = (x - 5)(x - 2)^6$ at $(1, -4)$.

A 23

B 8

C 40

D 25



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Q8: Find the first derivative of the function $y = x^4(4x + 9)^9$ at $x = -2$.

A -608

B -32

C 32

D 576

E 544



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Q9: Determine the derivative of $h(t) = (2t - 5)^{\frac{3}{5}}(-4t^2 + 4)^3$.

A $h'(t) = -8t(2t - 5)^{\frac{3}{5}}(-4t^2 + 4)^2 + \frac{2(-4t^2 + 4)^3}{(2t - 5)^{\frac{2}{5}}}$

B $h'(t) = 3(2t - 5)^{\frac{3}{5}}(-4t^2 + 4)^2 - \frac{6}{25}(-4t^2 + 4)^3$

C $h'(t) = -3(2t - 5)^{\frac{3}{5}}(-4t^2 + 4)^2 - \frac{6}{25}(-4t^2 + 4)^3$

D $h'(t) = -24t(2t - 5)^{\frac{3}{5}}(-4t^2 + 4)^2 + \frac{6(-4t^2 + 4)^3}{5(2t - 5)^{\frac{2}{5}}}$

E $h'(t) = 24t(2t - 5)^{\frac{3}{5}}(-4t^2 + 4)^2 + \frac{6(-4t^2 + 4)^3}{5(2t - 5)^{\frac{2}{5}}}$

Q10: Find $\frac{d}{dx} (5x\sqrt{2x^2 + 2})$ at $x = 1$.

- A 90
- B 70
- C -5
- D 15

Q11: Evaluate the first derivative of $y = 4x\sqrt{x + 6}$ at $(-2, -16)$.

- A 0
- B 8
- C 6
- D 10
- E 1

Q12: Determine the derivative of $F(t) = (-3t + 1)^5 (4t - 2)^{-3}$.

A $F'(t) = \frac{3(-3t + 1)^5}{(4t - 2)^4} + \frac{5(-3t + 1)^4}{(4t - 2)^3}$

B $F'(t) = \frac{12(-3t + 1)^5}{(4t - 2)^4} - \frac{15(-3t + 1)^4}{(4t - 2)^3}$

C $F'(t) = -\frac{3(-3t + 1)^5}{(4t - 2)^4} + \frac{5(-3t + 1)^4}{(4t - 2)^3}$

D $F'(t) = -\frac{12(-3t + 1)^5}{(4t - 2)^4} - \frac{15(-3t + 1)^4}{(4t - 2)^3}$

E $F'(t) = -\frac{12(-3t + 1)^5}{(4t - 2)^2} - \frac{15(-3t + 1)^4}{(4t - 2)^3}$

Q13: Determine the derivative of $g(x) = (-x + 1)^3 (3x^2 - x - 2)^6$.

A $g'(x) = -3(-x + 1)^2 (6x - 1) (3x^2 - x - 2)^5$

B $g'(x) = (-x + 1)^2 (-45x^2 + 45x) (3x^2 - x - 2)^6$

C $g'(x) = (-x + 1)^2 (-45x^2 - 45x) (3x^2 - x - 2)^5$

D $g'(x) = (-x + 1)^2 (-45x^2 + 27x) (3x^2 - x - 2)^5$

E $g'(x) = (-x + 1)^2 (-45x^2 + 45x) (3x^2 - x - 2)^5$

Q14: Find the first derivative of the function $y = (9x^2 - 7)\sqrt{2x + 1}$ at $x = 0$.

A -7

B 9

C 7

D $-\frac{63}{2}$

Q15: If $y = \sqrt{\frac{2x^3 + 1}{2x^3 - 1}}$, determine $\frac{dy}{dx}$.

A $-\frac{6x^2}{4x^6 + 1}$

B $-\frac{12x^2}{4x^6 - 1}$

C $-\frac{6x^2}{4x^6 - 1}$

D $-\frac{6x^2}{4x^9 - 1}$

Q16: Find the derivative of the function $y = \sqrt{\frac{x}{-5x - 2}}$.

A $y' = -\frac{1}{\sqrt{x}(-5x - 2)^{\frac{2}{3}}}$

B $y' = -\frac{1}{\sqrt{x}}(-5x - 2)^{\frac{2}{3}}$

C $y' = \frac{1}{\sqrt{x}(-5x - 2)^{\frac{3}{2}}}$

D $y' = -\frac{1}{\sqrt{x}}(-5x - 2)^{\frac{3}{2}}$

E $y' = -\frac{1}{\sqrt{x}(-5x - 2)^{\frac{3}{2}}}$

Q17: Evaluate $\frac{dy}{dx}$ at $(1, -1)$ if $y = -\frac{2x}{\sqrt{3x^2 + 1}}$.

A $-\frac{3}{2}$

B $\frac{9}{4}$

C -3

D $-\frac{1}{4}$

Q18: If $y = \frac{x}{\sqrt{a^2 + 5x^2}}$, find $x^9 \left(\frac{dy}{dx} \right)$.

A $a^2y^3x^6$

B ax^6y^3

C x^7y

D y^3

E x^9y^3

Q19: Evaluate $\frac{dy}{dx}$ at $x = 1$ if $y = (6x^2 - 2x - 3)^{-3} (x^2 - 2)^5$.

A 30

B 10

C -20

D 40

E 0



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Q20: Find the derivative of the function $U(y) = \left(\frac{y^3 - 4}{y^6 - 4}\right)^4$.

A $U'(y) = \frac{4(y^3 - 4)^3(-3y^8 + 24y^5 - 12y^2)}{(y^6 - 4)^5}$

B $U'(y) = \frac{4(y^3 - 4)^3(9y^8 - 24y^5 - 12y^2)}{(y^6 - 4)^5}$

C $U'(y) = \frac{4(y^3 - 4)^3(-3y^8 + 24y^5 - 12y^2)}{(y^6 - 4)^4}$

D $U'(y) = \frac{4(y^3 - 4)^3(9y^8 - 24y^5 - 12y^2)}{(y^6 - 4)^4}$

E $U'(y) = \frac{4(y^3 - 4)^4(-3y^8 + 24y^5 - 12y^2)}{(y^6 - 4)^6}$

Q21: Determine the derivative of the function $g(u) = \left(\frac{u^2 + 5}{u^2 - 1}\right)^4$.

A $g'(u) = -\frac{12u(u^2 + 5)^3}{(u^2 - 1)^5}$

B $g'(u) = 4\left(\frac{u^2 + 5}{u^2 - 1}\right)^3$

C $g'(u) = -\frac{48u(u^2 + 5)^3}{(u^2 - 1)^5}$

D $g'(u) = -\frac{24u^2(u^2 + 5)^3}{(u^2 - 1)^5}$

E $g'(u) = -\frac{48u(u^2 + 5)^3}{(u^2 - 1)^3}$

Q22: Determine $\frac{dy}{dx}$, given that $y = \left(\frac{x-23}{x+6}\right)^{-5}$.

A $-\frac{145(x+6)^6}{(x-23)^6}$

B $-\frac{145(x+6)^4}{(x-23)^6}$

C $-\frac{145(x+6)^3}{(x-23)^5}$

D $-\frac{17(x+6)^4}{(x-23)^6}$

E $\frac{85(x+6)^4}{(x-23)^6}$

Q23: Given that $y = \left(\frac{x^2-5}{x^2+5}\right)^n$, determine $\frac{dy}{dx}$.

A $\frac{20xy}{x^4+25}$

B $\frac{20nxy}{x^4+25}$

C $\frac{20xy}{x^4-25}$

D $\frac{20nxy}{x^4-25}$

Q24: Find the derivative of the function $H(r) = \frac{(-r^2 + 1)^5}{(-r + 4)^7}$.

A $H'(r) = \frac{(-r^2 + 1)^4 (3r^2 - 40r + 7)}{(-r + 4)^6}$

B $H'(r) = \frac{10r(-r^2 + 1)^4}{7(-r + 4)^8}$

C $H'(r) = \frac{(-r^2 + 1)^4 (3r^2 - 40r + 7)}{(-r + 4)^8}$

D $H'(r) = \frac{(-r^2 + 1)^4 (17r^2 - 40r - 7)}{(-r + 4)^8}$

E $H'(r) = \frac{(-r^2 + 1)^4 (17r^2 - 40r - 7)}{(-r + 4)^6}$

Q25: If $y = \frac{(x^2 + 4)^5}{(-4x^2 - 3)^4}$, find $\frac{dy}{dx}$.

A $\frac{2x(x^2 + 4)^4}{(4x^2 + 3)^5} (4x^2 - 49)$

B $\frac{2x(x^2 + 4)^4}{4x^2 + 3} (36x^2 + 79)$

C $-\frac{2x(x^2 + 4)^4}{(4x^2 + 3)^5} (36x^2 + 79)$

D $\frac{2x(x^2 + 4)^4}{(4x^2 + 3)^2} (4x^2 - 49)$