Worksheet: Comparing Rate of Growth of Functions

Q1: For the functions $f(x) = 2x^2$ and $g(x) = 4e^{2x}$, evaluate $\lim_{x\to\infty}\left(\frac{f(x)}{g(x)}\right)$ using l'Hôpital's rule.

A The limit does not exist.

B 0

C ∞

D 1

E $\frac{1}{4}$

Q2:

- Evaluate $\lim_{x\to\infty}\left(\frac{x^2}{e^x}\right)$ using l'Hôpital's rule.

A ∞

B 0

C 2

D $-\infty$

E 3

From the answer to the limit, what can you say about the growth rate of x^2 compared to e^x as $x \to \infty$?

A The growth rate of x^2 is greater than that of e^x as $x \to \infty$.

B The growth rate of x^2 is smaller than that of e^x as $x \to \infty$.

C The growth rate of x^2 is equal to that of e^x as $x \to \infty$.

D The growth rate of x^2 becomes ∞, while the growth rate of e^x becomes 0 as $x \to \infty$.

E The growth rate of x^2 becomes 0, while the growth rate of e^x becomes ∞ as $x \to \infty$.

Q3: If $\lim_{x\to\infty}\left(\frac{f(x)}{g(x)}\right) = 0$, what do you notice about the growth rate of $f(x)$ compared to $g(x)$ as $x \to \infty$?

A The growth rate of $f(x)$ is equal to that of $g(x)$ as $x \to \infty$.

B The growth rate of $f(x)$ becomes 0, while the growth rate of $g(x)$ becomes ∞ as $x \to \infty$.

C The growth rate of $f(x)$ is greater than that of $g(x)$ as $x \to \infty$.

D The growth rate of $f(x)$ becomes ∞, while the growth rate of $g(x)$ becomes 0 as $x \to \infty$.

E The growth rate of $f(x)$ is smaller than that of $g(x)$ as $x \to \infty$.

Q4: If $\lim_{x\to\infty}\left(\frac{f(x)}{g(x)}\right)=\infty$, what do you notice about the growth rate of $f(x)$ compared to $g(x)$ as $x\to\infty$?

A The growth rate of $f(x)$ becomes ∞, while the growth rate of $g(x)$ becomes 0 as $x\to\infty$.

B The growth rate of $f(x)$ is equal to that of $g(x)$ as $x\to\infty$.

C The growth rate of $f(x)$ is smaller than that of $g(x)$ as $x\to\infty$.

D The growth rate of $f(x)$ is greater than that of $g(x)$ as $x\to\infty$.

E The growth rate of $f(x)$ becomes 0, while the growth rate of $g(x)$ becomes ∞ as $x\to\infty$.

Q5: Compare the growth rate of the two functions $f(x)=(x+4)^2$ and $g(x)=\ln x$ using limits as $x\to\infty$.

A The growth rate of $f(x)$ is equal to the growth rate of $g(x)$.

B The growth rate of $f(x)$ becomes ∞, while the growth rate of $g(x)$ becomes 0 as $x\to\infty$.

C The growth rate of $f(x)$ is greater than the growth rate of $g(x)$.

D The growth rate of $f(x)$ becomes 0, while the growth rate of $g(x)$ becomes ∞ as $x\to\infty$.

E The growth rate of $f(x)$ is smaller than the growth rate of $g(x)$.

Q6: For the functions $f(x) = \ln(2x)$ and $g(x) = 4x^3$, evaluate $\lim_{x\to\infty}\left(\frac{f(x)}{g(x)}\right)$ using l'Hôpital's rule.

A ∞

B $\frac{1}{12}$

C $\frac{1}{6}$

D 0

E The limit does not exist.

Q7: Compare the growth rate of the two functions $f(x) = x^{10}$ and $g(x) = 10^x$ using limits as $x \to \infty$.

A The growth rate of $g(x)$ becomes 0, while the growth rate of $f(x)$ becomes ∞ as $x \to \infty$.

B The growth rate of $g(x)$ is greater than the growth rate of $f(x)$.

C The growth rate of $f(x)$ is greater than the growth rate of $g(x)$.

D The growth rate of $f(x)$ is equal to the growth rate of $g(x)$.

E The growth rate of $f(x)$ becomes 0, while the growth rate of $g(x)$ becomes ∞ as $x \to \infty$.

Q8: Compare the growth rate of the two functions $f(x) = e^x$ and $g(x) = \ln x$ using limits as $x \to \infty$.

A The growth rate of $\ln x$ becomes 0, while the growth rate of e^x becomes ∞ as $x \to \infty$.

B The growth rate of $\ln x$ is greater than the growth rate of e^x.

C The growth rate of e^x is greater than the growth rate of $\ln x$.

D The growth rate of e^x becomes 0, while the growth rate of $\ln x$ becomes ∞ as $x \to \infty$.

E The growth rate of e^x is equal to the growth rate of $\ln x$.

Q9: Compare the growth rate of the two functions $f(x) = x^2$ and $g(x) = \frac{1}{x}$ using limits as $x \to \infty$.

A The growth rate of $f(x)$ becomes 0, while the growth rate of $g(x)$ becomes ∞ as $x \to \infty$.

B The growth rate of $f(x)$ is equal to the growth rate of $g(x)$.

C The growth rate of $f(x)$ is smaller than the growth rate of $g(x)$.

D The growth rate of $f(x)$ is greater than the growth rate of $g(x)$.

E The growth rate of $f(x)$ becomes ∞, while the growth rate of $g(x)$ becomes 0 as $x \to \infty$.

Q10: Consider the functions $f(x) = x^{50}$ and $g(x) = 2^X$.

Evaluate $\lim_{x\to\infty}\left(\frac{f(x)}{g(x)}\right)$ using l'Hôpital's rule.

A The limit does not exist.

B 1

C $\frac{1}{2}$

D 0

E ∞

Evaluate $\lim_{x\to\infty}\left(\frac{g(x)}{f(x)}\right)$ using l'Hôpital's rule.

A The limit does not exist.

B 1

C 2

D ∞

E 0

What do you get from the two results about the growth rates of x^{50} and 2^x as $x \to \infty$?

A The growth rate of x^{50} becomes 0, while the growth rate of 2^x becomes ∞ as $x \to \infty$.

B The growth rate is the same for both as $x \to \infty$.

C The growth rate of 2^x becomes 0, while the growth rate of x^{50} becomes ∞ as $x \to \infty$.

D The growth rate of 2^x is greater than that of x^{50} as $x \to \infty$.

E The growth rate of x^{50} is greater than that of 2^x as $x \to \infty$.