

# Worksheet: Stochastic Matrix



**Q1:** On any given day, an animal eats either cheese, grapes, or lettuce. The food the animal eats is influenced by the food it ate the previous day:

- If it eats cheese today, tomorrow it will eat either lettuce or grapes, with equal probability, but not cheese.
  - If it eats grapes today, tomorrow it will eat grapes with a probability of  $\frac{1}{10}$ , cheese with a probability of  $\frac{4}{10}$ , or lettuce with a probability of  $\frac{5}{10}$ .
  - If it eats lettuce today, tomorrow it will eat grapes with a probability of  $\frac{4}{10}$  or cheese with a probability of  $\frac{6}{10}$ , but not lettuce.
- Represent these probabilities in the matrix  $P$ , where the columns, in order, represent the events of eating cheese, grapes, or lettuce tomorrow; and the rows, in order, represent the events of eating cheese, grapes, or lettuce today.

A  $P = \begin{bmatrix} 0 & 0.4 & 0.6 \\ 0.4 & 0.1 & 0.4 \\ 0.6 & 0.5 & 0 \end{bmatrix}$

B  $P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.4 & 0.1 & 0.5 \\ 0.6 & 0.4 & 0 \end{bmatrix}$

C  $P = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.4 & 0.1 & 0.5 \\ 0.6 & 0.4 & 0 \end{bmatrix}$

D  $P = \begin{bmatrix} 1 & 0.5 & 0.5 \\ 0.4 & 0.1 & 0.5 \\ 0.6 & 0.5 & 1 \end{bmatrix}$

E  $P = \begin{bmatrix} 0 & 0.4 & 0.6 \\ 0.4 & 0.1 & 0.5 \\ 0.6 & 0.4 & 0 \end{bmatrix}$

▶ Letting  $Q = P^2$ , what is the interpretation of the entry  $Q_{1,2}$ ?

- A the expected amount of cheese the animal eats
- B the probability that the animal eats cheese given that two days earlier the animal ate grapes
- C the probability that the animal eats grapes given that two days earlier the animal ate cheese
- D the expected number of grapes the animal eats
- E the probability that the animal does not eat lettuce

**Q2:** What does it mean for a matrix  $A = (a_{ij})$  to be stochastic?

A  $A$  has nonnegative entries and  $\sum_j a_{ij} \geq 1$ .

B  $A$  has nonnegative entries and  $\sum_j a_{ij} = 1$ .

C  $A$  has negative entries and  $\sum_j a_{ij} > 1$ .

D  $\sum_j a_{ij} = 1$

**Q3:** Under which of the following conditions is the  $n \times n$  matrix  $A = (a_{ij})$  a Markov matrix?

A  $a_{ij} \geq 0$  for all  $i, j$  and  $\sum_i a_{ij} = 1$

B  $a_{ij} \geq 0$  for all  $i, j$  and  $\sum_i a_{ij} = 0$

C  $a_{ij} > 0$  for all  $i, j$  and  $\sum_i a_{ij} = 1$

D  $a_{ij} \geq 0$  for all  $i, j$  and  $\sum_i a_{ij} \geq 1$