Worksheet: The Differentiability of a Function

Q1: Let

$$f(x) = \begin{cases} 5a + bx^2 & \text{if } x < -2, \\ 5 & \text{if } x = -2, \\ ax - 3b & \text{if } x > -2. \end{cases}$$

Determine the values of a and b so that f is continuous at $x = -2$. What can be said of the differentiability of f at this point?

A $a = -4, b = 1$, differentiable at $x = -2$

B $a = 5, b = -5$, differentiable at $x = -2$

C $a = -4, b = 1$, not differentiable at $x = -2$

D $a = 5, b = -5$, not differentiable at $x = -2$

Q2: Discuss the continuity and differentiability of the function f at $x = 0$ given

$$f(x) = \begin{cases} -9x - 6 & \text{if } x < 0, \\ x^2 - 9x - 6 & \text{if } x \geq 0. \end{cases}$$

A The function is continuous and differentiable at $x = 0$.

B The function is not continuous, so it is not differentiable at $x = 0$.

C The function is continuous but not differentiable at $x = 0$.

D The function is not continuous but differentiable at $x = 0$.

Q3: Discuss the differentiability of a function f at $x = -4$ given

$$f(x) = \begin{cases} 8x + 7 & \text{if } x < -4, \\ 2x + 5 & \text{if } x \geq -4. \end{cases}$$

A $f(x)$ is differentiable at $x = -4$ because $f'(-4^+) = f'(-4^-)$.

B $f(x)$ is differentiable at $x = -4$ because f is continuous at $x = -4$.

C $f(x)$ is not differentiable at $x = -4$ because $f(-4)$ is undefined.

D $f(x)$ is not differentiable at $x = -4$ because $f'(-4^+) \neq f'(-4^-)$.

Q4: Discuss the differentiability of the function $f(x)$ at $x = 1$ given $f(x) = (6x - 6)\,|6x - 6|$.

A The function is differentiable at $x = 1$ as $f(x)$ is continuous at that point.

B The function is not differentiable at $x = 1$ as $f(x)$ is discontinuous at that point.

C The function is differentiable at that point as $f'(1^-) = f'(1^+)$.

D The function is not differentiable at that point as $f'(1^-) \neq f'(1^+)$.

Q5: Discuss the differentiability of the function f at $x = 1$ given

$$f(x) = \begin{cases} 2x + 8 & \text{if } x < 1, \\ x^2 + 9 & \text{if } x \geq 1. \end{cases}$$

A $f(x)$ is discontinuous but differentiable at $x = 1$ because $f'(1^+) = f'(1^-)$.

B $f(x)$ is not differentiable at $x = 1$ because f is discontinuous at $x = 1$.

C $f(x)$ is not differentiable at $x = 1$ because $f(1)$ is undefined.

D $f(x)$ is differentiable at $x = 1$.

E $f(x)$ is continuous but not differentiable at $x = 1$ because $f'(1^+) \neq f'(1^-)$.

Q6: Suppose

$$f(x) = \begin{cases} x^2 - 7x + 5 & \text{if } x \le -8, \\ 3x^2 + 4x - 4 & \text{if } x > -8. \end{cases}$$

What can be said of the differentiability of f at $x = -8$?

A. The function $f(x)$ is not differentiable at $x = -8$ because $f'(-8^-) \neq f'(-8^+)$.

B. The function $f(x)$ is differentiable at $x = -8$ because $f(-8)$ is undefined.

C. The function $f(x)$ is not continuous but differentiable at $x = -8$ because $f'(-8^-) = f'(-8^+)$.

D. The function $f(x)$ is not differentiable at $x = -8$ because f is discontinuous at $f(-8)$.

E. The function $f(x)$ is differentiable at $x = -8$ as $\lim_{x \to -8^-} f(x) = \lim_{x \to -8^+} f(x)$ but is not continuous.

Q7: What can be said of the differentiability of $f(x) = \sqrt{x^2 + 4x + 4}$ at $x = -2$?

A. $f(x)$ is not differentiable at $x = -2$ because $f(-2)$ is undefined.

B. $f(x)$ is differentiable at $x = -2$.

C. $f(x)$ is not differentiable at $x = -2$ because f is discontinuous at that point.

D. $f(x)$ is continuous but not differentiable at $x = -2$ because $f'(-2^+) \neq f'(-2^-)$.

Q8: Discuss the differentiability of the function $f(x) = -4x + \frac{1}{x}$ at $x = -7$.

A The function is differentiable at $x = -7$ because $f(-7)$ exists.

B The function is not differentiable at $x = -7$ because $f(x)$ is not continuous at that point.

C The function is differentiable at $x = -7$ because $f'(-7)$ exists.

D The function is not differentiable at $x = -7$ because $f'(-7)$ does not exist.

Q9: Let

$$f(x) = \begin{cases} -4c + mx & \text{if } x < 1, \\ cx^2 - 4m & \text{if } x \geq 1. \end{cases}$$

If $f(1) = 12$ and f is continuous at $x = 1$, determine the values of m and c. What can be said of the differentiability of f at this point?

A $m = -4$, $c = -4$, not differentiable at $x = 1$

B $m = -12$, $c = -6$, differentiable at $x = 1$

C $m = -4$, $c = -4$, differentiable at $x = 1$

D $m = -12$, $c = -6$, not differentiable at $x = 1$

Q10: Find the values of a and b given the function f is differentiable at $x = -1$ where

$$f(x) = \begin{cases} 9x + 5 & \text{if } x < -1, \\ ax^2 + bx - 4 & \text{if } x \geq -1. \end{cases}$$

A $a = -4, b = 1$

B $a = -10, b = -8$

C $a = -18, b = 0$

D $a = -9, b = -9$

Q11: Find the values of a and b given the function f is differentiable at $x = 1$ where

$$f(x) = \begin{cases} -x^2 + 4 & \text{if } x \leq 1, \\ -2ax - b & \text{if } x > 1. \end{cases}$$

A $a = 1, b = -5$

B $a = 4, b = 5$

C $a = -1, b = 5$

D $a = 3, b = 6$

Q12: Let

$$f(x) = \begin{cases} ax + 8x^2 - 4 & \text{if } x < 1, \\ -4 & \text{if } x = 1, \\ a + bx & \text{if } x > 1. \end{cases}$$

Determine the values of a and b so that f is continuous at $x = 1$. What can be said of the differentiability of f at this point?

A $a = -10$, $b = 6$, f is not differentiable at $x = 1$

B $a = -8$, $b = 4$, f is not differentiable at $x = 1$

C $a = -8$, $b = 4$, f is differentiable at $x = 1$

D $a = -10$, $b = 6$, f is differentiable at $x = 1$

Q13: Discuss the differentiability of the function f at $x = 6$, given

$$f(x) = \begin{cases} -7x & \text{if } -8 \leq x \leq -2, \\ 8x^2 + 5x & \text{if } -2 < x \leq 6. \end{cases}$$

A $f(x)$ is discontinuous but differentiable at $x = 6$ because $f'(6^+) = f'(6^-)$.

B $f(x)$ is not differentiable at $x = 6$ because $f(x)$ is discontinuous at $x = 6$.

C $f(x)$ is not differentiable at $x = 6$ because $f(6)$ is undefined.

D $f(x)$ is continuous but not differentiable at $x = 6$ because $f'(6^+) \neq f'(6^-)$.

E $f(x)$ is differentiable at $x = 6$.

Q14: Given that

$$f(x) = \begin{cases} 8x - 8 & \text{if } x < -2, \\ ax^3 & \text{if } x \geq -2. \end{cases}$$

is a continuous function, find the value of a. What can be said of the differentiability of f at $x = -2$?

A $a = 3$, f is differentiable at $x = -2$

B $a = -3$, f is not differentiable at $x = -2$

C $a = -3$, f is differentiable at $x = -2$

D $a = 3$, f is not differentiable at $x = -2$

Q15: Suppose

$$f(x) = \begin{cases} -6x - 4 & \text{if } x \leq -1, \\ 3x^2 & \text{if } x > -1. \end{cases}$$

What can be said of the differentiability of f at $x = -1$?

A The function $f(x)$ is not differentiable at $x = -1$.

B The function $f(x)$ is differentiable at $x = -1$ as $\lim_{x \to -1^-} f(x) \neq \lim_{x \to -1^+} f(x)$ but is not continuous.

C The function $f(x)$ is not differentiable at $x = -1$ because $f(x)$ is continuous at $f(-1)$.

D The function $f(x)$ is continuous but not differentiable at $x = -1$ because $f'(-1^-) \neq f'(-1^+)$.

E The function $f(x)$ is not differentiable at $x = -1$ because $f(-1)$ is undefined.

Q16: Discuss the differentiability of the function f at $x = 1$ given

$$f(x) = \begin{cases} 8x^2 - 7x + 3 & \text{if } -2 \leq x < 1, \\ 4x & \text{if } 1 \leq x \leq 3. \end{cases}$$

A $f(x)$ is continuous but not differentiable at $x = 1$ because $f'(1^+) \neq f'(1^-)$.

B $f(x)$ is not differentiable at $x = 1$ because $f(x)$ is discontinuous at $x = 1$.

C $f(x)$ is discontinuous but differentiable at $x = 1$ because $f'(1^+) = f'(1^-)$.

D $f(x)$ is differentiable at $x = 1$.

Q17: Suppose

$$f(x) = \begin{cases} x^2 - 9 & \text{if } x \leq 4, \\ x + 3 & \text{if } x > 4. \end{cases}$$

What can be said of the differentiability of f at $x = 4$?

A The function is continuous and differentiable at $x = 4$ because $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4)$.

B The function is not continuous, so it is not differentiable at $x = 4$.

C The function is continuous but not differentiable at $x = 4$ because $f'(4^-) \neq f'(4^+)$.

D The function is not continuous but differentiable at $x = 4$ because $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4)$.

Q18: Suppose

$$f(x) = \begin{cases} -8x^2 + 7 & \text{if } x \leq 0, \\ a - 2x^2 & \text{if } x > 0. \end{cases}$$

Find the value of a such that the function f is continuous at $x = 0$, and then discuss the differentiability of f at $x = 0$.

A $a = -7$, differentiable at $x = 0$

B $a = -7$, not differentiable at $x = 0$

C $a = 7$, differentiable at $x = 0$

D $a = 7$, not differentiable at $x = 0$

Q19: Find the values of a and b and discuss the differentiability of the function f at $x = -1$ given f is continuous and

$$f(x) = \begin{cases} 9x^2 + ax + 4 & \text{if } x < -1, \\ 11 & \text{if } x = -1, \\ a + bx & \text{if } x > -1. \end{cases}$$

A $a = -2$, $b = 9$, and $f(x)$ is differentiable at $x = -1$.

B $a = 2$, $b = 9$, and $f(x)$ is not differentiable at $x = -1$.

C $a = 2$, $b = -9$, and $f(x)$ is not differentiable at $x = -1$.

D $a = -2$, $b = -9$, and $f(x)$ is differentiable at $x = -1$.

E $a = 8$, $b = 4$, and $f(x)$ is not differentiable at $x = -1$.

Q20: What can be said of the differentiability of $f(x) = 9x^2 + 8x + 7$ at $x = -2$?

A $f(x)$ is not differentiable at $x = -2$ because $f(x)$ is not continuous.

B $f(x)$ is differentiable at $x = -2$.

C $f(x)$ is not differentiable at $x = -2$ because $f'(-2)$ does not exist.

D $f(x)$ is not differentiable at $x = -2$ because $f(-2)$ is undefined.

Q21: Discuss the differentiability of the function f at $x = -4$ given

$$f(x) = \begin{cases} -6x^2 + 7x - 4 & \text{if } -4 \leq x < -1, \\ -2x & \text{if } -1 \leq x \leq 1. \end{cases}$$

A The function $f(x)$ is continuous, but not differentiable at $x = -4$ because $f'(-4^+) \neq f'(-4^-)$.

B The function $f(x)$ is discontinuous, but differentiable at $x = -4$ because $f'(-4^+) = f'(-4^-)$.

C The function $f(x)$ is not differentiable at $x = -4$ because $f(x)$ is discontinuous at $x = -4$.

D The function $f(x)$ is differentiable at $x = -4$.

Q22: Suppose

$$f(x) = \begin{cases} 4x - 7 & \text{if } x < 1, \\ 2x^2 - 5 & \text{if } x \geq 1. \end{cases}$$

What can be said of the differentiability of f at $x = 1$?

- A The function $f(x)$ is discontinuous but differentiable at $x = 1$ because $f'(1^+) = f'(1^-)$.
- B The function $f(x)$ is differentiable at $x = 1$.
- C The function $f(x)$ is not differentiable at $x = 1$ because f is discontinuous at $x = 1$.
- D The function $f(x)$ is not differentiable at $x = 1$ because $f(1)$ is undefined.
- E The function $f(x)$ is continuous but not differentiable at $x = 1$ because $f'(1^+) \neq f'(1^-)$.

Q23: Let

$$f(x) = \begin{cases} 12x - 6 & \text{if } x < 2, \\ ax^2 + 6 & \text{if } x \geq 2. \end{cases}$$

Determine the value of a so that f is continuous at $x = 2$. What can be said of the differentiability of f at this point?

- A $a = 3$, not differentiable at $x = 2$
- B $a = 3$, differentiable at $x = 2$
- C $a = 12$, not differentiable at $x = 2$
- D $a = 12$, differentiable at $x = 2$

Q24: Discuss the differentiability of the function f at $x = -8$ given

$$f(x) = \begin{cases} 6x & \text{if } -9 \leq x \leq -8, \\ 6x^2 + 6x & \text{if } -8 < x \leq 5. \end{cases}$$

A $f(x)$ is not differentiable at $x = -8$ because $f(x)$ is discontinuous at $x = -8$.

B $f(x)$ is continuous but not differentiable at $x = -8$ because $f'(-8^+) \neq f'(-8^-)$.

C $f(x)$ is differentiable at $x = -8$ because $f(x)$ is continuous at $x = -8$.

D $f(x)$ is discontinuous but differentiable at $x = -8$ because $f'(-8^+) = f'(-8^-)$.

E $f(x)$ is not differentiable at $x = -8$ because $f(-8)$ is undefined.

Q25: Suppose

$$f(x) = \begin{cases} x^2 - 15 & \text{if } x \leq 1, \\ 2x - 16 & \text{if } x > 1. \end{cases}$$

What can be said of the differentiability of f at $x = 1$?

A The function is not continuous, so it is not differentiable at $x = 1$.

B The function is continuous and differentiable at $x = 1$ because $f'(1^-) = f'(1^+)$.

C The function is continuous but not differentiable at $x = 1$ because $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$.

D The function is not continuous but differentiable at $x = 1$ because $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$.