

Worksheet: First-Order Linear Differential Equations



Q1: Solve the differential equation $\frac{dy}{dx} + y = x$.

A $y = x + 1 + Ce^{-x}$

B $y = \frac{x^2 e^x}{2} + Ce^{-x}$

C $y = x - 1 + Ce^x$

D $y = x - 1 + Ce^{-x}$

E $y = \frac{x^2 e^{-x}}{2} + Ce^{-x}$

Q2: Solve the differential equation $x \frac{dy}{dx} + y = x \ln x$, where $x > 0$, subject to the condition $y(1) = 0$.

A $y = \frac{x}{2} \ln x + \frac{x}{4} + \frac{1}{4x}$

B $y = \frac{x}{2} \ln x - \frac{x}{4} + \frac{1}{2x}$

C $y = \frac{x}{4} \ln x - \frac{x}{4} + \frac{1}{4x}$

D $y = \frac{x}{2} \ln x - \frac{x}{4} + \frac{1}{4x}$

E $y = x \ln x - \frac{x}{4} + \frac{1}{4x}$

Q3: Solve the differential equation $x^2 \frac{dy}{dx} + xy = 1$, where $x > 0$, subject to the condition $y(1) = 2$.

A $y = \frac{\ln x - \ln 2 + 2}{x}$

B $y = \frac{\ln x + 2}{2x}$

C $y = \frac{-1}{x^2 + 3}$

D $y = \frac{\ln x + 2}{x}$

E $y = \frac{\ln x}{x}$

Q4: Solve the differential equation $x \frac{dy}{dx} = y + x^2 \sin x$ subject to the condition $y(\pi) = 0$.

A $y = x \cos x - x$

B $y = x \cos x + x$

C $y = x \cos x$

D $y = -x \cos x - x$

E $y = -x \cos x + x$

Q5: Solve the differential equation $t \frac{du}{dt} = t^2 + 3u$ subject to the condition $u(2) = 4$.

A $u = -t + t^3$

B $u = -t^2 + t^3$

C $u = -t^2 - t^3$

D $u = \frac{t^2}{5} + \frac{128}{5t^3}$

E $u = t^2$

Q6: Solve the differential equation $t^2 \frac{dy}{dt} + 3ty = \sqrt{1+t^2}$, where $t > 0$.

A $y = \frac{1}{3} (1+t^2)^{\frac{1}{2}} t^{-3} + Ct^{-3}$

B $y = \frac{1}{3} (1+t^2)^{\frac{3}{2}} t^{-3} + Ct^{-3}$

C $y = \frac{1}{3} (1+t)^{\frac{3}{2}} t^{-3} + Ct^{-3}$

D $y = \frac{1}{3} (1+t)^2 t^{-3} + Ct^{-3}$

E $y = (1+t^2)^{\frac{3}{2}} t^{-3} + Ct^{-3}$

Q7: Solve the differential equation $x\frac{dy}{dx} + y = \sqrt{x}$.

A $y = \frac{2}{5}\sqrt{x^3} + \frac{C}{x}$

B $y = \frac{2\sqrt{x}}{3} + \frac{C}{x}$

C $y = \frac{2\sqrt{x}}{3} + C$

D $y = \frac{2}{5}\sqrt{x^3} + Cx$

E $y = \frac{\sqrt{x}}{2} + \frac{C}{x}$

Q8: Solve the differential equation $2x\frac{dy}{dx} + y = 2\sqrt{x}$.

A $y = x + \frac{C}{x}$

B $y = \sqrt{x^3} + \frac{C}{\sqrt{x}}$

C $y = \sqrt{x} + \frac{C}{\sqrt{x}}$

D $y = 1 + \frac{C}{\sqrt{x}}$

E $y = x + \frac{C}{\sqrt{x}}$

Q9: Solve the differential equation $(x^2 + 1) \frac{dy}{dx} + 3x(y - 1) = 0$ subject to the condition $y(0) = 2$.

A $y = 1 + \frac{1}{(x^2 + 1)^{\frac{3}{2}}}$

B $y = 1 + \frac{1}{(x^2 + 1)^{\frac{1}{2}}}$

C $y = 1 - \frac{1}{(x^2 + 1)^{\frac{1}{2}}}$

D $y = 1 - \frac{5\sqrt{5}}{(x^2 + 1)^{\frac{3}{2}}}$

E $y = 3 + \frac{1}{(x^2 + 1)^{\frac{3}{2}}}$

Q10: Is the differential equation $\frac{dr}{dt} + t \cos r = e^{-t}$ linear?

A yes

B no

Q11: Solve the differential equation $x \frac{dy}{dx} - 2y = x^2$, where $x > 0$.

- A $\frac{1}{4}x^2 + \frac{C}{x^2}$
- B $y = -x^2(\ln x + C)$
- C $y = -\frac{1}{2}x^2(\ln x + C)$
- D $y = x^2(\ln x + C)$
- E $y = x^2(x + C)$

Q12: Solve the differential equation $x^2 \frac{dy}{dx} + 2xy = \ln x$ subject to the condition $y(1) = 2$.

- A $y = \frac{x \ln x - x + 1}{x^2}$
- B $y = \frac{x \ln x + 3}{x^2}$
- C $y = \frac{\ln x - x + 3}{x^2}$
- D $y = \frac{\ln^2 x + 4}{2x}$
- E $y = \frac{x \ln x - x + 3}{x^2}$

Q13: Solve the differential equation $4x^3y + x^4 \frac{dy}{dx} = \sin^3 x$.

A $y = \frac{-\cos^2 x + 3 \cos x + C}{3x^4}$

B $y = \frac{\cos^3 x - \cos x + C}{3x^4}$

C $y = \frac{\cos^3 x + 3 \cos x + C}{3x^4}$

D $y = \frac{\cos^3 x - 3 \cos x + C}{3x^4}$

E $y = \frac{-\cos^3 x + 3 \cos x + C}{3x^4}$

Q14: Solve the differential equation $t^3 \frac{dy}{dt} + 3t^2y = \cos t$ subject to the condition $y(\pi) = 0$.

A $y = \frac{t \sin t + \cos t + 1}{t^3}$

B $y = \frac{\sin t + 1}{t^3}$

C $y = \frac{\sin t}{t^3}$

D $y = -\frac{\sin t + 1}{t^3}$

E $y = -\frac{\sin t}{t^3}$

Q15: Is the differential equation $ue^t = t + \sqrt{t}\frac{du}{dt}$ linear?

A yes

B no

Q16: Solve the differential equation $t \ln t \frac{dr}{dt} + r = te^t$, where $t > 0$.

A $r = \frac{e^{-t} + C}{\ln t}$

B $r = \frac{e^t + C}{\ln 2t}$

C $r = \frac{1 + C}{\ln t}$

D $r = \frac{e^{-t} + C}{2 \ln t}$

E $r = \frac{e^t + C}{\ln t}$

Q17: Solve the differential equation $\frac{dy}{dx} - y = e^x$.

A $y = (x + C)e^{2x}$

B $y = (e^x + C)e^x$

C $y = (x + C)e^x$

D $y = (x + C)e^{-x}$

E $y = -(x + C)e^x$

Q18: Is the differential equation $\frac{dy}{dx} - x = y \tan x$ linear?

A yes

B no