Worksheet: Root Test

Q1: Consider the series $\sum_{n=0}^{\infty} a_n$, where $a_n = \left(\frac{n^3 + n^2 - n + 3}{3n^3 + 6n^2 + 1}\right)^{2n}$.

- Calculate $\lim_{n\to\infty} |a_n|^{\frac{1}{n}}$.

A 1

B $\frac{1}{9}$

C ∞

D 0

E $\frac{1}{3}$

- Hence, determine whether the series converges or diverges.

A It diverges.

B It converges.

Q2: Consider the series $\sum_{n=0}^{\infty} a_n$, where $a_n = \frac{(n+1)^n}{6^{2n}}$.

- Calculate $\lim_{n\to\infty} |a_n|^{\frac{1}{n}}$.

A $\frac{1}{36}$

B 0

C 6

D ∞

E $\frac{1}{6}$

- Hence, determine whether the series converges or diverges.

A It converges.

B It diverges.

Q3: A series $\sum_{n=0}^{\infty} a_n$ satisfies $\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = 1$.
What can we conclude about the convergence of the series?

A We cannot conclude anything.

B The series converges conditionally.

C The series diverges.

D The series converges absolutely.

Q4: Consider the series $\sum_{n=3}^{\infty} \left(\frac{2-n}{3n+1}\right)^n$.

- Is this an alternating series?

A no

B yes

- Is this series absolutely convergent, conditionally convergent, or divergent?

A conditionally convergent

B absolutely convergent

C divergent

Q5: Consider the series $1 + \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{3}} + \frac{1}{2\sqrt{4}} + \cdots$, where the term $a_n = \frac{1}{2\sqrt{n}}$.

- What is $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$?

A $\frac{1}{2}$

B -2

C -1

D 2

E 1

What is $\lim_{n\to\infty} \sqrt[n]{\frac{1}{2\sqrt{n}}}$?

A $\frac{1}{2}$

B -1

C -2

D 2

E 1

Use l'Hopital's rule to determine the value of the limit $\lim_{n\to\infty} \frac{A \ln n}{\sqrt{n}}$, where $A > 0$ is a constant.

A $\frac{1}{2}$

B 1

C $\frac{1}{\sqrt{2}}$

D 2

E 0

What does the previous result tell you about the values of $\sqrt{n}$ and $\log_2 n^2$, where $n \geq 1$ is an integer?

A It tells us that $\sqrt{n} < \log_2 n^2$ for all large values of n.

B It tells us that $\sqrt{n}$ and $\log_2 n^2$ are both zero if n is large enough.

C It tells us nothing.

D It tells us that $\sqrt{n} > \log_2 n^2$ for all values of n.

E It tells us that $\sqrt{n} > \log_2 n^2$ for all large values of n.

Using the comparison test, is this series convergent or divergent?

A Divergent

B Convergent