Worksheet: Operations on Power Series

Q1: Suppose that $\sum_{n=0}^{\infty} a_n x^n$ is a power series whose interval of convergence is $(-3, 3)$ and that $\sum_{n=0}^{\infty} b_n x^n$ is a power series whose interval of convergence is $(-5, 5)$.

- Find the interval of convergence of the series $\sum_{n=0}^{\infty} (a_n x^n - b_n x^n)$.

A $(-8, 8)$

B $(-5, 3)$

C $(-3, 3)$

D $(-3, 5)$

E $(-5, 5)$

- Find the interval of convergence of the series $\sum_{n=0}^{\infty} b_n 2^n x^n$.

A $(-10, 10)$

B $(-3, 3)$

C $\left(-\frac{5}{2}, \frac{5}{2}\right)$

D $(-5, 5)$

E $(-6, 6)$

Q2: Use partial fractions to find the power series of the function $f(x) = \dfrac{3}{(x-2)(x+1)}$.

A $\sum_{n=0}^{\infty}\left[(-1)^n + \left(\frac{1}{2}\right)^n\right]x^n$

B $\sum_{n=0}^{\infty}\left[(2)^{n+1} - (-1)^{n+1}\right]x^n$

C $\sum_{n=0}^{\infty}\left[(-1)^n - \left(\frac{1}{2}\right)^n\right]x^n$

D $\sum_{n=0}^{\infty}\left[(-1)^{n+1} + (2)^{n+1}\right]x^n$

E $\sum_{n=0}^{\infty}\left[(-1)^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right]x^n$

Q3: Consider the power series $\sum_{n=0}^{\infty} 3^n x^n$.

▸ Find the function f represented by this series.

A $f(x) = \dfrac{1}{1-3x}$

B $f(x) = \dfrac{1}{3+x}$

C $f(x) = \dfrac{1}{3-x}$

D $f(x) = \dfrac{1}{3x-1}$

E $f(x) = \dfrac{1}{1+3x}$

▸ Determine the interval of convergence of the series.

A $\left(-\frac{1}{3}, \frac{1}{3}\right)$

B $(-1, 1)$

C $(-3, 3)$

D $\left(-1, \frac{1}{3}\right)$

E $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Q4: Consider the functions $f(x) = \sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$ and $g(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^n}{n!}$.

▸ Find the power series of $\frac{1}{2}[f(x) + g(x)]$.

A $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{(2n+1)!}$

B $\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{2n}}{2n!}$

C $\sum_{n=0}^{\infty} \frac{(x-1)^{2n+1}}{(2n+1)!}$

D $\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{2n+1}}{2n!}$

E $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{2n!}$

▸ Find the power series of $\frac{1}{2}\left[f(x) - g(x)\right]$.

A $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{(2n+1)!}$

B $\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{2n}}{2n!}$

C $\sum_{n=0}^{\infty} \frac{(x-1)^{2n}}{2n!}$

D $\sum_{n=0}^{\infty} (-1)^n \frac{(x-1)^{2n+1}}{2n!}$

E $\sum_{n=0}^{\infty} \frac{(x-1)^{2n+1}}{(2n+1)!}$

Q5: Let $f(x) = \frac{1}{(x-1)(x-2)}$.

▸ Construct a power series for the function $f(x)$.

A $\sum_{n=0}^{\infty} \left[1 - \left(\frac{1}{2}\right)^n\right] x^n$

B $\sum_{n=0}^{\infty} \left[1 - \left(\frac{1}{2}\right)^{n+1}\right] x^n$

C $\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n + 1\right] x^n$

D $\sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] x^n$

E $\sum_{n=0}^{\infty} \left[1 + \left(\frac{1}{2}\right)^{n+1}\right] x^n$

▸ Find the interval of convergence of the power series.

A $\left(-1, \frac{3}{2}\right)$

B $(-1, 1)$

C $(-2, 2)$

D $\left(-\frac{3}{2}, 1\right)$

E $\left(-\frac{3}{2}, \frac{3}{2}\right)$

Q6: Use partial fractions to find the power series of the function $f(x) = \frac{3}{(x^2+1)(x^2+4)}$.

A $\sum_{n=0}^{\infty} (-1)^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right] x^{2n}$

B $\sum_{n=1}^{\infty} \left[1 - \left(\frac{1}{4}\right)^{n+1}\right] x^{2n}$

C $\sum_{n=0}^{\infty} \left[1 - \left(\frac{1}{4}\right)^{2n}\right] x^{n}$

D $\sum_{n=1}^{\infty} (-1)^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right] x^{2n}$

E $\sum_{n=0}^{\infty} \left[1 - \left(\frac{1}{4}\right)^{n+1}\right] x^{2n}$

Q7: Consider the functions $f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{2n}}{(2n)!}$ and $g(x) = \sum_{n=0}^{\infty} \frac{(2x)^{2n+1}}{(2n+1)!}$.

Find the power series of $f(x) + g(x)$.

A $\sum_{n=0}^{\infty} \frac{(2x)^n}{(n)}$

B $\sum_{n=0}^{\infty} \frac{x^n}{(n)!}$

C $\sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{n+1}}{(n+1)}$

D $\sum_{n=0}^{\infty} \frac{(2x)^n}{(n)!}$

E $\sum_{n=0}^{\infty} (-1)^n \frac{(2x)^n}{(n)!}$

Find the power series of $f(x) - g(x)$.

A $\sum_{n=0}^{\infty} \frac{(2x)^n}{(n)}$

B $\sum_{n=0}^{\infty} \frac{x^n}{(n)!}$

C $\sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{n+1}}{(n+1)}$

D $\sum_{n=0}^{\infty} (-1)^n \frac{(2x)^n}{(n)!}$

E $\sum_{n=0}^{\infty} \frac{(2x)^n}{(n)!}$

Q8: Multiply the series $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$ by itself to construct a series for $\frac{1}{(1+x)^2}$. Write the answer in sigma notation.

A $\sum_{n=0}^{\infty} x^{2n}$

B $\sum_{n=0}^{\infty} (-1)^n (n+1)x^n$

C $\sum_{n=0}^{\infty} (n+1)x^n$

D $\sum_{n=0}^{\infty} n(-1)^n x^{n+1}$

E $\sum_{n=0}^{\infty} (-1)^n x^{n+1}$

Q9: Consider the power series $f(x) = \sum_{n=0}^{\infty} \frac{1}{4^n} x^n$.

▸ Find the function f represented by this series.

A $f(x) = \frac{4}{4-x}$

B $f(x) = \frac{1}{x+4}$

C $f(x) = \frac{1}{4-4x}$

D $f(x) = \frac{4}{x-4}$

E $f(x) = \frac{4}{4+x}$

Determine the interval of convergence of the series.

A $(-4, 4)$

B $\left(-\frac{1}{4}, 1\right)$

C $\left(-4, \frac{1}{4}\right)$

D $\left(-\frac{1}{4}, \frac{1}{4}\right)$

E $(-1, 1)$

Let $g(x) = f(-x)$. Find $f(x) + g(x)$ in sigma notation.

A $2 \sum_{n=0}^{\infty} \frac{1}{4^{2n}} x^{2n}$

B $8 \sum_{n=0}^{\infty} (-1)^n \frac{1}{4^n} x^{2n}$

C $\sum_{n=0}^{\infty} (-1)^n \frac{1}{4^n} x^n$

D $\sum_{n=0}^{\infty} \frac{1}{2^n} x^n$

E $8 \sum_{n=0}^{\infty} \frac{1}{4^n} x^n$

Q10: Find $\sum_{n=0}^{\infty} x^n \cdot \sum_{n=0}^{\infty} (2x)^n$.

A $\sum_{n=0}^{\infty} (2^n + 1)\, x^{n+1}$

B $\sum_{n=0}^{\infty} \left(2^{n+1} + 1\right) x^n$

C $\sum_{n=0}^{\infty} (2^n)\, x^n$

D $\sum_{n=0}^{\infty} \left(2^{n+1} - 1\right) x^n$

E $\sum_{n=0}^{\infty} \left(2^{n+1} - 1\right) x^{n+1}$