

Worksheet: Applications of Triple Integrals



Q1: Find the center of mass of the solid $S = \{(x, y, z): 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1 - x - y\}$ with the given density function $\rho(x, y, z) = 1$.

A $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

B $(4, 4, 4)$

C $\left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right)$

D $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$

E $\left(\frac{1}{8}, \frac{1}{8}, \frac{1}{8}\right)$

Q2: Find the center of mass of the solid $S = \{(x, y, z): x \geq 0, y \geq 0, z \geq 0, x^2 + y^2 + z^2 \leq a^2\}$ with the given density function $\rho(x, y, z) = 1$.

A $\left(\frac{3a}{16}, \frac{3a}{16}, \frac{3a}{16}\right)$

B $\left(\frac{a}{2}, \frac{a}{2}, \frac{a}{2}\right)$

C $\left(\frac{8}{3a}, \frac{8}{3a}, \frac{8}{3a}\right)$

D $\left(\frac{3a}{8}, \frac{3a}{8}, \frac{3a}{8}\right)$

E $\left(\frac{2a}{3}, \frac{2a}{3}, \frac{2a}{3}\right)$

Q3: Find the center of mass of the solid $S = \{(x, y, z): 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1\}$ with the given density function $\rho(x, y, z) = xyz$.

A $\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$

B $(1, 1, 1)$

C $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

D $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

E $\left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right)$

Q4: Find the center of mass of the solid $S = \{(x, y, z): 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1\}$ with the given density function $\rho(x, y, z) = x^2 + y^2 + z^2$.

A $\left(\frac{7}{12}, \frac{7}{12}, \frac{7}{12}\right)$

B $(1, 1, 1)$

C $\left(\frac{5}{7}, \frac{5}{7}, \frac{5}{7}\right)$

D $\left(\frac{12}{7}, \frac{12}{7}, \frac{12}{7}\right)$

E $\left(\frac{5}{12}, \frac{5}{12}, \frac{5}{12}\right)$

Q5: Find the center of mass of the solid $S = \{(x, y, z): z \geq 0, x^2 + y^2 + z^2 \leq a^2\}$ with the given density function $\rho(x, y, z) = x^2 + y^2 + z^2$.

A $\left(0, 0, \frac{12}{5a}\right)$

B $\left(0, 0, \frac{5a}{12}\right)$

C $\left(0, 0, \frac{5a}{6}\right)$

D $\left(0, 0, \frac{5a}{3}\right)$

E $\left(0, 0, \frac{5a}{8}\right)$

Q6: Let a , b , and c be real numbers selected randomly from the interval $(0, 1)$. What is the probability that the equation $ax^2 + bx + c = 0$ has at least one real solution for x ? Rounding the value to four decimal places.

A 0.7456

B 0.2544

C 0.4766

D 0.3451

E 0.5234