Worksheet: Conservation of Energy in Circular Motion

Q1: A uniform circular hoop of mass 1.2 kg and radius 0.6 m is rotating in a horizontal plane about a smooth vertical axis through point A of its circumference.

- Calculate the kinetic energy of the hoop when it is rotating at 5 rad/s.

A 10.8 J

B 4.32 J

C 2.16 J

D 5.4 J

E 21.6 J

- A particle of mass 0.4 kg is now fixed to the hoop at point B, where AB is a diameter. The hoop continues to rotate at 5 rad/s. Calculate the total kinetic energy of the hoop and the particle.

A 18 J

B 11.52 J

C 9.36 J

D 12.6 J

E 28.8 J

Q2: A uniform circular disc of mass 1.5 kg and radius 0.4 m is free to rotate in a horizontal plane about a fixed smooth vertical axis through the centre of the disc. Particles of masses 0.5 kg, 0.8 kg, and 1.2 kg are attached to points on the circumference of the disc. The loaded disc is rotating at 100 revolutions per minute. Calculate the angular momentum of the loaded disc to the nearest decimal place.

A $14.0\ \text{kg}\cdot\text{m}^2/\text{s}$

B $7.2\ \text{kg}\cdot\text{m}^2/\text{s}$

C $2.7\ \text{kg}\cdot\text{m}^2/\text{s}$

D $5.4\ \text{kg}\cdot\text{m}^2/\text{s}$

E $37.7\ \text{kg}\cdot\text{m}^2/\text{s}$

Q3: A uniform square lamina has sides of length 0.8 m and a mass of 4 kg. It is free to rotate about a fixed axis which coincides with one of its sides. Calculate the change of angular momentum when the angular speed of the lamina is increased from 2 rad/s to 5 rad/s.

A $5.12\ \text{N}\cdot\text{m}\cdot\text{s}$

B $4.27\ \text{N}\cdot\text{m}\cdot\text{s}$

C $1.71\ \text{N}\cdot\text{m}\cdot\text{s}$

D $10.24\ \text{N}\cdot\text{m}\cdot\text{s}$

E $2.56\ \text{N}\cdot\text{m}\cdot\text{s}$

Q4: A smooth solid hemisphere with a radius of 14.8 m rests with its flat face on a horizontal surface. The centre of the flat circular face is the point O. Suppose a particle of mass 8 kg starts sliding from rest at the highest point of the hemisphere. How far is the particle from O when it hits the surface? Give your answer correct to one decimal place. Take $g = 9.8$ m/s^2.

A 16.1 m

B 5.6 m

C 9.9 m

D 17.3 m

E 16.6 m

Q5: A marble of a mass 238 g is attached to one end of a light elastic string whose natural length is 0.2 m and modulus of elasticity is 11 N. Suppose that the other end of the string is attached to a fixed point O on a smooth horizontal surface and the marble moves in a horizontal circular path at a constant angular speed of 3 rad/s. Determine the length of the string rounded to the nearest hundredth. Take $g = 9.8$ m/s^2.

A 0.21 m

B 0.20 m

C 0.04 m

D 0.10 m

Q6: A light rod AB of length 2.5 m, fixed to the wall at B and attached to a ball of mass 2.4 kg at A, is free to rotate in a vertical plane about B. Given that the ball was held at rest such that AB was horizontal and then it was released, determine the tension in the rod as the ball passes through the lowest point in its path. Take $g = 9.8$ m/s^2.

A 23.52 N

B 141.12 N

C 70.56 N

D 49.44 N

E 47.04 N

Q7: A bead of mass 0.3 kg is attached to one end of a light rod of length 0.54 m. The other end of the rod is fixed at a point O, about which the rod can freely rotate in a vertical plane. The rod is held at an angle of 45° to the upward vertical and released from rest. Taking $g = 9.8$ m/s^2, find the speed of the bead as it passes through the lowest point on its path. Give your answer in m/s, correct to one decimal place.

A 1.2 m/s

B 3.0 m/s

C 1.8 m/s

D 2.2 m/s

E 4.3 m/s

Q8: A light rod, AB, of length 2.9 m is fixed to a wall at A and attached to a ball of mass 4.4 kg at B. The rod is free to rotate in a vertical plane about B. The ball was held at rest such that A was vertically above B and then it was slightly nudged. Determine the speed of the ball as it passes through the lowest point in its path, rounding your answer to the nearest tenth. Consider the acceleration due to gravity to be 9.8 m/s^2.

A 7.5 m/s

B 15.8 m/s

C 10.7 m/s

D 5.3 m/s

E 11.2 m/s

Q9: A bead of mass 0.063 kg is attached to one end of a light rod of length 0.22 m. The other end of the rod is fixed at a point O about which the rod can freely rotate in a vertical plane. The rod is held horizontally and released. Taking $g = 9.8$ m/s^2, find the bead's speed as it passes through the lowest point on its path. Give your answer in metres per second correct to one decimal place.

A 0.5 m/s

B 2.1 m/s

C 1.7 m/s

D 1.5 m/s

E 0.1 m/s

Q10: A smooth solid hemisphere, with radius 11 m and centre O, rests with its flat face on a horizontal surface. A ball, with centre B, of mass 17 kg, rests on the hemisphere's highest point. The ball is barely pushed. When OB turns through an angle, θ, to the upward vertical, and the ball is still on the surface of the hemisphere, the normal reaction of the hemisphere on the ball is R. Find an expression for R in terms of g and θ, where g is the acceleration due to gravity.

A $561g\sin\theta - 374g$

B $51g\cos\theta - 34g$

C $561g\cos\theta - 374g$

D $51g\sin\theta - 34g$

E $374g - 17g\cos\theta$

Q11: A man on a motorcycle is going round a traffic circle of radius 28 m. The coefficient of friction between the motorcycle's tyres and the road is $\frac{5}{7}$. Find the greatest angular speed, in radians per second, at which the motorcycle can move around the traffic circle without sliding. Consider the acceleration due to gravity to be $g = 9.8$ m/s^2.

A 0.5 rad/s

B 14 rad/s

C 19.6 rad/s

D 0.354 rad/s

E 9.899 rad/s

Q12: A particle is held at a point A on a smooth solid hemisphere of radius 3 m and centre O. The particle is released and slides down the hemisphere under gravity before leaving the hemisphere at point B. Given that OA makes an angle of 26° with the upward vertical, find the angle that OB makes with the upward vertical. Take $g = 9.8$ m/s^2 and give your answer correct to one decimal place.

A 72.6°

B 81.6°

C 30.9°

D 73.7°

E 53.2°

Q13: A bead B of mass 0.9 kg is attached to one end of a light rod of length 0.3 m. The other end of the rod is fixed at a point O, about which the rod can freely rotate in a vertical plane. The bead was at rest vertically below O when it was pushed horizontally at a speed of 14 m/s. Taking $g = 9.8$ m/s^2, find the bead's speed when it passed the point vertically above O. Give your answer in metres per second correct to one decimal place.

A 13.6 m/s

B 9.6 m/s

C 184.2 m/s

D 13.8 m/s

E 14.4 m/s

Q14: A rough horizontal disc is rotating at a constant angular speed of 7 rad/s about a vertical axis through its centre. A rock resting on this disc is on the point of slipping. Taking $g = 9.8$ m/s^2, determine the coefficient of friction between the rock and the disc given that the rock lies 11 cm away from the centre of the disc.

A 0.921

B 0.55

C 0.157

D 0.45

E 0.079

Q15: A particle resting at the highest point of a smooth sphere is barely pushed such that it slides down the sphere's surface, moving at a speed of $\frac{\sqrt{11gr}}{4}$, where g is the acceleration due to gravity and r is the radius of the sphere. Determine the vertical distance travelled by the particle before it left the sphere's surface.

A $\frac{5r}{16}$

B $\frac{33r}{64}$

C $\frac{11r}{32}$

D $\frac{11r}{24}$

E $\frac{5r}{32}$

Q16: A smooth sphere of radius 5.1 m is fixed to a horizontal surface. A particle of mass 2 kg at the highest point of the sphere slides down its surface from rest, leaves the sphere, and finally hits the horizontal surface. Taking $g = 9.8$ m/s^2, find the velocity of the particle when it hits the surface. Express the magnitude of the velocity, v, in metres per second correct to one decimal place, and the direction of the velocity, θ, as the angle made with the horizontal to the nearest degree.

A $v = 14.1$ m/s, $\theta = 74°$

B $v = 14.1$ m/s, $\theta = 16°$

C $v = 5.8$ m/s, $\theta = 16°$

D $v = 13.6$ m/s, $\theta = 16°$

E $v = 13.6$ m/s, $\theta = 74°$

Q17: A turn on a racing track follows a horizontal circular arc of radius 74 m. The track at this turn is banked at an angle of 19° to help cars go round it at speed without slipping. Given that the coefficient of friction between a car's tyres and the track is 0.8 and taking $g = 9.8$ m/s^2, find the maximum speed the car can go around this turn without slipping. Give your answer in metres per second correct to one decimal place.

A 35.6 m/s

B 25.5 m/s

C 33.8 m/s

D 24.9 m/s

E 42.4 m/s

Q18: A rough horizontal disc is rotating about a vertical axis through its centre. A stone resting on this disc, at a distance of 0.3 m from its centre, is on the point of slipping. Given that the coefficient of friction between the stone and the disc is 0.1 and taking $g = 9.8$ m/s^2, find the angular speed of the disc in radians per second, giving your answer correct to one decimal place.

A 5.4 rad/s

B 1.8 rad/s

C 3.3 rad/s

D 2.6 rad/s

E 29.4 rad/s