

Worksheet: Improper Integrals: Discontinuous Integrands



Q1: The integral $\int_0^1 r \ln r \, dr$ is convergent. What does it converge to?

A $-\frac{1}{4}$

B $\frac{1}{4}$

C 0

D $-\frac{1}{2}$

E $\frac{1}{2}$

Q2: Determine whether the integral $\int_0^1 \frac{1}{x} \, dx$ is convergent or divergent.

A Divergent

B Convergent

Q3: Determine whether the integral $\int_{-1}^2 \frac{x}{(x+1)^2} \, dx$ is convergent or divergent.

A divergent

B convergent

Q4: The integral $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ is convergent. What does it converge to?

A $\frac{\pi}{2}$

B $-\frac{\pi}{2}$

C $\frac{\pi}{4}$

D 0

E $-\frac{\pi}{4}$

Q5: The integral $\int_0^5 \frac{1}{\sqrt[3]{5-x}} dx$ is convergent. What does it converge to?

A $-\frac{3}{4}5^{-\frac{4}{3}}$

B $\frac{3}{4}5^{-\frac{4}{3}}$

C $-\frac{3}{2}5^{\frac{2}{3}}$

D $\frac{2}{3}5^{\frac{2}{3}}$

E $\frac{3}{2}5^{\frac{2}{3}}$

Q6: The integral $\int_{-2}^{14} \frac{1}{\sqrt[4]{x+2}} dx$ is convergent. What does it converge to?

A -32

B -2

C 6

D $-\frac{32}{3}$

E $\frac{32}{3}$

Q7: Determine whether the integral $\int_0^1 \frac{e^{\frac{1}{x}}}{x^3} dx$ is convergent or divergent.

A convergent

B divergent

Q8: The integral $\int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^3} dx$ is convergent. What does it converge to?

A $-\frac{2}{e}$

B $-\frac{1}{e}$

C $-2e$

D e

E $\frac{2}{e}$

Q9: Determine whether the integral $\int_0^{\frac{\pi}{2}} \tan^2 \theta \, d\theta$ is convergent or divergent.

A divergent

B convergent

Q10: The integral $\int_0^{\frac{\pi}{2}} \frac{\cos \theta}{\sqrt{\sin \theta}} \, d\theta$ is convergent. What does it converge to?

A $\frac{2}{3}$

B -2

C 2

D $-\frac{2}{3}$

E $-\frac{1}{2}$

Q11: Determine whether the integral $\int_{-2}^3 \frac{1}{x^4} \, dx$ is convergent or divergent.

A convergent

B divergent

Q12: Determine whether the integral $\int_0^5 \frac{w}{w-2} dw$ is convergent or divergent.

A convergent

B divergent

Q13: Determine whether the integral $\int_0^4 \frac{1}{x^2 - x - 2} dx$ is convergent or divergent.

A divergent

B convergent

Q14: The integral $\int_0^9 \frac{1}{\sqrt[3]{x-1}} dx$ is convergent. What does it converge to?

A -1

B $-\frac{15}{2}$

C 2

D $-\frac{1}{2}$

E $\frac{9}{2}$

Q15: Consider the integral $\int_0^1 x^p \ln x dx$.

► Find all possible values of p for which the integral is convergent.

A $p \geq -1$

B $p = -1$

C $p > -1$

D $p \leq -1$

E $p < -1$

► Evaluate the integral for those values of p .

A $\frac{1}{(p+1)^2}$

B $\frac{1}{p+1}$

C $-\frac{1}{(p+1)^2}$

D $-\frac{1}{(p-1)^2}$

E $-\frac{1}{p-1}$

Q16: Consider the integral $\int_0^1 \frac{1}{x^p} dx$.

► Find all possible values of p for which the integral is convergent.

A $p = 1$

B $p \leq 1$

C $p < 1$

D $p > 1$

E $p \geq 1$

► Evaluate the integral for those values of p .

A $1 + p$

B $\frac{1}{1 + p}$

C $\frac{1}{1 - p}$

D $\frac{1}{p}$

E $1 - p$

Q17: Consider the integral $\int_a^b \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$. Which of the following is a correct characterization of this integral?

- A The integral is undefined if $a < 0$, $b > 0$ and $|a| \neq |b|$.
- B The integral is undefined only if $a = 0$ or $b = 0$.
- C The integral is undefined only if a or b is an integer multiple of π .
- D The integral is defined for all finite values of a and b .
- E The integral is undefined if $a < 0$ and $b > 0$.

Q18: Is $\int_0^{\frac{\pi}{4}} \tan x dx$ an improper integral?

- A yes
- B no

Q19: Is $\int_0^{\pi} \tan x dx$ an improper integral?

- A No
- B Yes