Worksheet: Separable Differential Equations

Q1: Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} + y = 1$.

A $y = xe^{-x} + \mathrm{C}e^{-x}$

B $y = 1 + \mathrm{C}e^{-x}$

C $y = 1 + \mathrm{C}e^{x}$

D $y = x + \mathrm{C}e^{-x}$

E $y = x + \mathrm{C}e^{x}$

Q2: Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = -5x^2y^2$.

A $y = \frac{3}{5x^3 + \mathrm{C}}$ or $y = 0$

B $y = -\frac{3}{5x^3 + \mathrm{C}}$ or $y = 0$

C $y = -\frac{1}{15x^3 + \mathrm{C}}$ or $y = 0$

D $y = \frac{1}{5x^3 + \mathrm{C}}$ or $y = 0$

E $y = \frac{1}{15x^3 + \mathrm{C}}$ or $y = 0$

Q3: Solve the differential equation $\frac{\mathrm{d}H}{\mathrm{d}R} = \frac{RH^2\sqrt{1+R^2}}{\ln H}$.

A $-\frac{\ln H}{H} - \frac{1}{H} = \frac{2}{3}\left(1+R^2\right)^{\frac{3}{2}} + \mathrm{C}$

B $-\frac{\ln H}{H} - \frac{1}{H} = \frac{1}{2}\left(1+R^2\right)^{\frac{3}{2}} + \mathrm{C}$

C $-\frac{\ln H}{H} - \frac{1}{H} = \frac{1}{3}\left(1+R^2\right)^{\frac{3}{2}} + \mathrm{C}$

D $-\frac{\ln H}{H} + \frac{1}{H} = \frac{1}{3}\left(1+R^2\right)^{\frac{3}{2}} + \mathrm{C}$

E $\frac{\ln H}{H} + \frac{1}{H} = \frac{1}{3}\left(1+R^2\right)^{\frac{3}{2}} + \mathrm{C}$

Q4: Solve the differential equation $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{t\sec\theta}{\theta e^{t^2}}$.

A $\theta\sin\theta + \cos\theta = -\frac{e^{-t^2}}{2} + \mathrm{C}$

B $\theta\sin\theta = -\frac{e^{-t^2}}{2} + \mathrm{C}$

C $-\theta\sin\theta - \cos\theta = -\frac{e^{-t^2}}{2} + \mathrm{C}$

D $\theta\sin\theta + \cos\theta = e^{-t^2} + \mathrm{C}$

E $-\theta\sin\theta - \cos\theta = e^{-t^2} + \mathrm{C}$

Q5: Find a relation between y and x, given that $xyy' = x^2 - 5$.

A $y^2 = 2x^2 - 10\ln|x| + \mathrm{C}$

B $y^2 = x^2 - 5\ln|x| + \mathrm{C}$

C $y^2 = 2x^2 - 10\ln x + \mathrm{C}$

D $y^2 = \frac{x^2}{2} - 5\ln|x| + \mathrm{C}$

E $y^2 = x^2 - 10\ln|x| + \mathrm{C}$

Q6: Solve the following differential equation: $\frac{\mathrm{d}p}{\mathrm{d}t} = t^2p - 5p + t^2 - 5$.

A $p = \mathrm{K}e^{\frac{1}{3}t^3 - 5t} + 1$

B $p = \mathrm{K}e^{t^3 - 5t} - 1$

C $p = \mathrm{K}e^{t^3 - 5t} + 1$

D $p = \mathrm{K}e^{\frac{1}{3}t^3 - 5t} - 1$

E $p = \mathrm{K}e^{\frac{1}{3}t^3 - 5} - 1$

Q7: Solve the following differential equation: $(e^y - 5)\, y' = 2 + \cos x$.

A $e^y - 5 = 2x - \sin x + \mathrm{C}$

B $e^y - 5y = 2x - \sin x + \mathrm{C}$

C $e^y - 5y = x + \sin x + \mathrm{C}$

D $e^y - 5y = 2x + \sin x + \mathrm{C}$

E $e^y - 5 = 2x + \sin x + \mathrm{C}$

Q8: Solve the differential equation $y' + xe^y = 0$.

A $y = \ln\left(2x^2 + \mathrm{C}\right)$

B $y = \ln\left(\frac{x^2}{2} + \mathrm{C}\right)$

C $y = -\ln\left(x^2 + \mathrm{C}\right)$

D $y = -\ln\left(2x^2 + \mathrm{C}\right)$

E $y = -\ln\left(\frac{x^2}{2} + \mathrm{C}\right)$

Q9: Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = -5x\sqrt{y}$.

A $\sqrt{y} = -5x^2 + \mathrm{C}$ or $y = 0$

B $y = \left(-\frac{5x^2}{4} + \mathrm{C}\right)^2$ or $y = 0$

C $y = \left(-\frac{5x^2}{2} + \mathrm{C}\right)^2$ or $y = 0$

D $\sqrt{y} = -\frac{5x^2}{2} + \mathrm{C}$ or $y = 0$

E $\sqrt{y} = -\frac{5x^2}{4} + \mathrm{C}$ or $y = 0$

Q10: Solve the following differential equation by separating it:

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = (1 - y^2)^{\frac{1}{2}}.$$

A $y = \ln(\sin|x| + \mathrm{C})$

B $y = \sin(\ln|x| + \mathrm{C})$

C $y = \ln(\cos|x| + \mathrm{C})$

D $y = \cos(\ln|x| + \mathrm{C})$

Q11: Find a 1-parameter family of solutions for the differential equation $yy' = (y+1)^2, y \neq -1$.

A $-\dfrac{1}{y+1} + \ln|y+1| = x + c$

B $\dfrac{1}{y+1} + \ln|y+1| = x + c$

C There is no solution.

D $\dfrac{1}{y+1} - \ln|y+1| = x + c$

Q12: Find the implicit solution to the following differential equation:

$$\sin(y)\frac{dy}{dx} - \cos(x) = 0.$$

A $\cos(y) + \sin(x) = C$

B $\sin(y) + \cos(x) = C$

C $\cos(y) + \csc(x) = C$

D $\cos(y) + \sec(x) = C$

Q13: Which of the following is a solution of $x + yy' = 0$ defined for all $-4 < x < 4$?

A $y = \sqrt{16 - x^2}$

B $y = \sqrt{4 - x^2}$

C $y = \sqrt{16 + x^2}$

D $y = \sqrt{4 + x^2}$

Q14: Find a relation between u and t given that $\dfrac{\mathrm{d}u}{\mathrm{d}t} = \dfrac{1+t^4}{ut^2+u^4t^2}$.

A $\dfrac{u^5}{5} + \dfrac{u^2}{2} = -\dfrac{1}{t} + \dfrac{t^3}{3} + \mathrm{C}$

B $\dfrac{u^5}{5} + \dfrac{u^2}{2} = \dfrac{1}{t} + \dfrac{t^3}{3} + \mathrm{C}$

C $u^5 + u^2 = \dfrac{1}{t} + \dfrac{t^3}{3} + \mathrm{C}$

D $\dfrac{u^5}{5} + \dfrac{u^2}{2} = -\dfrac{1}{t} + t^3 + \mathrm{C}$

E $u^5 + u^2 = -\dfrac{1}{t} + \dfrac{t^3}{3} + \mathrm{C}$

Q15: Solve the differential equation $\dfrac{\mathrm{d}z}{\mathrm{d}t} + e^{2t+2z} = 0$.

A $z = \dfrac{1}{2}\ln\left(e^{2t} + \mathrm{C}\right)$

B $z = -\dfrac{1}{2}\ln\left(2e^{2t} + \mathrm{C}\right)$

C $z = -\dfrac{1}{2}\ln\left(e^{2t} + \mathrm{C}\right)$

D $z = -\dfrac{1}{2}\ln\left(\dfrac{e^{2t}}{2} + \mathrm{C}\right)$

E $z = -\dfrac{1}{2}\ln\left(e^{2t} + \mathrm{C}\right)$

Q16: Solve the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} + 3x^2y = 6x^2$.

A $y = 2 - \mathrm{C}e^{-x^3}$

B $y = 6 + \mathrm{C}e^{-x^3}$

C $y = 2x^3e^{-x^3} + \mathrm{C}$

D $y = 2x^3e^{-x^3} + \mathrm{C}e^{-x^3}$

E $y = 2 + \mathrm{C}e^{-x^3}$