

# Worksheet: Representing Rational Functions Using Power Series



**Q1:** Let us consider  $g(x) = \frac{x}{3-x}$ .

► Find a power series representation for  $g(x)$ .

A  $g(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{3}$

B  $g(x) = \sum_{n=0}^{\infty} \frac{x^n}{3^{n+1}}$

C  $g(x) = \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$

D  $g(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{3^n}$

E  $g(x) = \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^{n+1}$

► Find its radius of convergence.

A  $|x| < 1$

B  $|x| > 3$

C  $|x| > 1$

D  $|x| < \frac{1}{3}$

E  $|x| < 3$

**Q2:** Let us consider  $f(x) = \frac{1}{1+x^2}$ .

► Find a power series representation for  $f(x)$ .

A  $f(x) = \sum_{n=0}^{\infty} x^{2n}$

B  $f(x) = \sum_{n=0}^{\infty} (-1)^n x^n$

C  $f(x) = \sum_{n=0}^{\infty} (-1)^n x^{2+n}$

D  $f(x) = \sum_{n=0}^{\infty} x^{2+n}$

E  $f(x) = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

► Find its interval of convergence.

A  $|x| < 0$

B  $|x| > 1$

C  $|x| > 0$

D  $|x| < \frac{1}{2}$

E  $|x| < 1$

**Q3:** Consider  $h(x) = \frac{4x^3}{1+x^5}$ .

► Find a power series representation for  $h(x)$ .

A  $h(x) = \sum_{n=0}^{\infty} 4(-1)^n x^{5n}$

B  $h(x) = \sum_{n=0}^{\infty} 4(-1)^n x^{5n+3}$

C  $h(x) = \sum_{n=0}^{\infty} 4(-1)^n x^{n+3}$

D  $h(x) = \sum_{n=0}^{\infty} 4(-x)^{5n+3}$

E  $h(x) = \sum_{n=0}^{\infty} (-1)^n x^{5n+3}$

► Find its interval of convergence.

A  $|x| < 0$

B  $|x| < 1$

C  $|x| > 0$

D  $|x| < 4$

E  $|x| > 1$

**Q4:** Use a power series to represent  $\frac{x}{1+x^2}$ .

A  $\sum_{n=0}^{\infty} (-1)^n x^{2n+1}$

B  $\sum_{n=0}^{\infty} x^{2n+1}$

C  $\sum_{n=0}^{\infty} (-1)^n x^{2n}$

D  $\sum_{n=0}^{\infty} x^{2n}$

E  $\sum_{n=0}^{\infty} x^{2n+2}$

**Q5:** Consider the function  $f(x) = \frac{1}{2+x}$ .

► Find the power series for  $f(x)$ .

A  $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} (x)^n$

B  $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+2} (x)^n$

C  $\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^{n+1} (-x)^n$

D  $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} (-x)^n$

E  $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n (-x)^n$

► Identify its interval of convergence.

A  $[-2, 2]$

B  $(0, 1]$

C  $(-1, 1)$

D  $(-2, 2)$

E  $(-\infty, \infty)$

**Q6:** Consider the function  $f(x) = \frac{1}{1 - 9x^2}$ .

► Find the power series for  $f(x)$ .

A  $\sum_{n=0}^{\infty} (9)^{2n} (x)^{2n}$

B  $\sum_{n=0}^{\infty} (3)^{2n} (x)^{2n}$

C  $\sum_{n=0}^{\infty} (3)^n (x)^{2n}$

D  $\sum_{n=0}^{\infty} (-3)^{2n} (x)^{2n}$

E  $\sum_{n=0}^{\infty} (3)^{2n} (-x)^{2n}$

► Identify its interval of convergence.

A  $\left[-\frac{1}{3}, \frac{1}{3}\right]$

B  $\left(-\frac{1}{3}, \frac{1}{3}\right)$

C  $\left[-\frac{1}{9}, \frac{1}{9}\right]$

D  $(-1, 1)$

E  $\left(-\frac{1}{9}, \frac{1}{9}\right)$

**Q7:** Consider the function  $f(x) = \frac{x^2}{9 + x^2}$  with the center  $a = 0$ . Find the power series for  $f(x)$ .

A  $\sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{3}\right)^{2n}$

B  $\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^{2n+2}$

C  $\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^{2n}$

D  $\sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{3}\right)^{2n+2}$

E  $\sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{3}\right)^{2n+1}$

**Q8:** Consider the function  $f(x) = \frac{1}{x^2 - 2x + 2}$  with the center  $a = 1$ .

Find the power series for  $f(x)$ .

A  $\sum_{n=0}^{\infty} (-1)^n (x - 1)^{2n}$

B  $\sum_{n=0}^{\infty} (-1)^n (x)^{2n}$

C  $\sum_{n=0}^{\infty} (-1)^n (x - 1)^n$

D  $\sum_{n=0}^{\infty} (-1)^n (x - 1)^{2n+1}$

E  $\sum_{n=0}^{\infty} (x - 1)^{2n}$

**Q9:** Consider the function  $f(x) = \frac{2}{x}$  with the center  $a = 1$ .

Find the power series for  $f(x)$ .

A  $\sum_{n=0}^{\infty} (1 - x)^n$

B  $2 \sum_{n=0}^{\infty} (1 - x)^n$

C  $2 \sum_{n=0}^{\infty} (-1)^n (1 - x)^n$

D  $\sum_{n=0}^{\infty} (-1)^n (1 - x)^n$

E  $2 \sum_{n=0}^{\infty} (1 - x)^{2n}$

**Q10:** Consider the function  $f(x) = \frac{x}{1 - 9x^2}$ .

► Find the power series for  $f(x)$ .

A  $\sum_{n=0}^{\infty} 3^{2n} x^{2n+1}$

B  $\sum_{n=0}^{\infty} 9^{2n} (-x)^{2n+1}$

C  $\sum_{n=0}^{\infty} (-3)^{2n} x^{2n+1}$

D  $\sum_{n=0}^{\infty} 3^n x^{2n}$

E  $\sum_{n=0}^{\infty} 9^{2n} x^{2n+1}$

► Identify its interval of convergence.

A  $\left(-\frac{1}{3}, \frac{1}{3}\right)$

B  $\left(-\frac{1}{9}, \frac{1}{9}\right)$

C  $\left[-\frac{1}{9}, \frac{1}{9}\right]$

D  $(-1, 1)$

E  $\left[-\frac{1}{3}, \frac{1}{3}\right]$