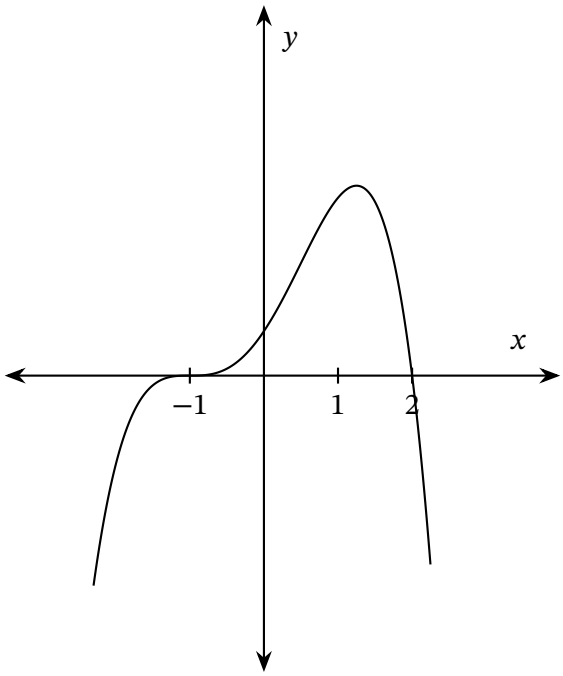
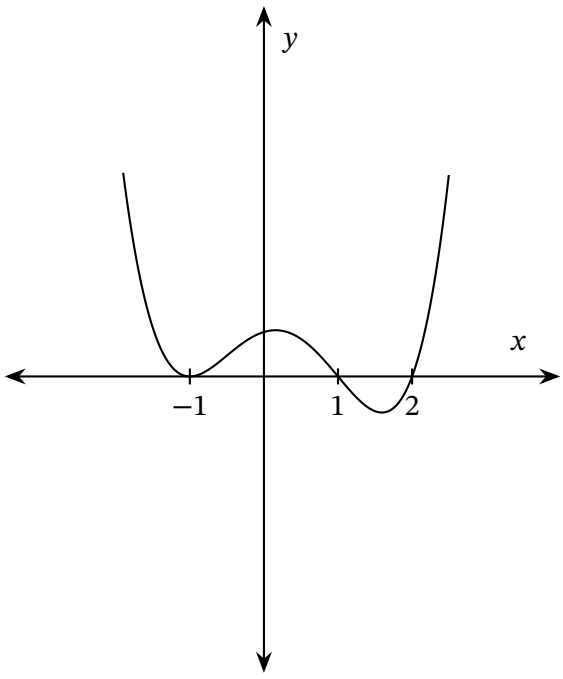
Worksheet: Graphing Using Derivatives

Q1: Which of the following is the graph of $f(x) = -(x+1)^3(x-2)$?

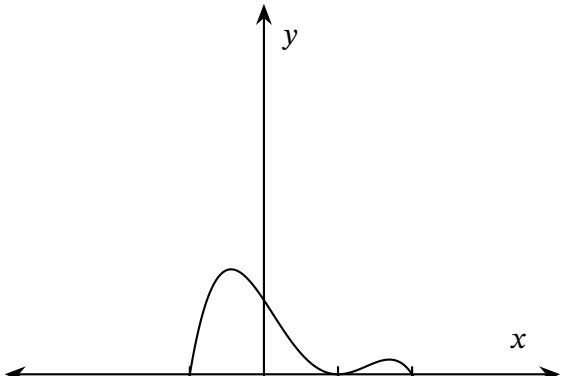
Question Video



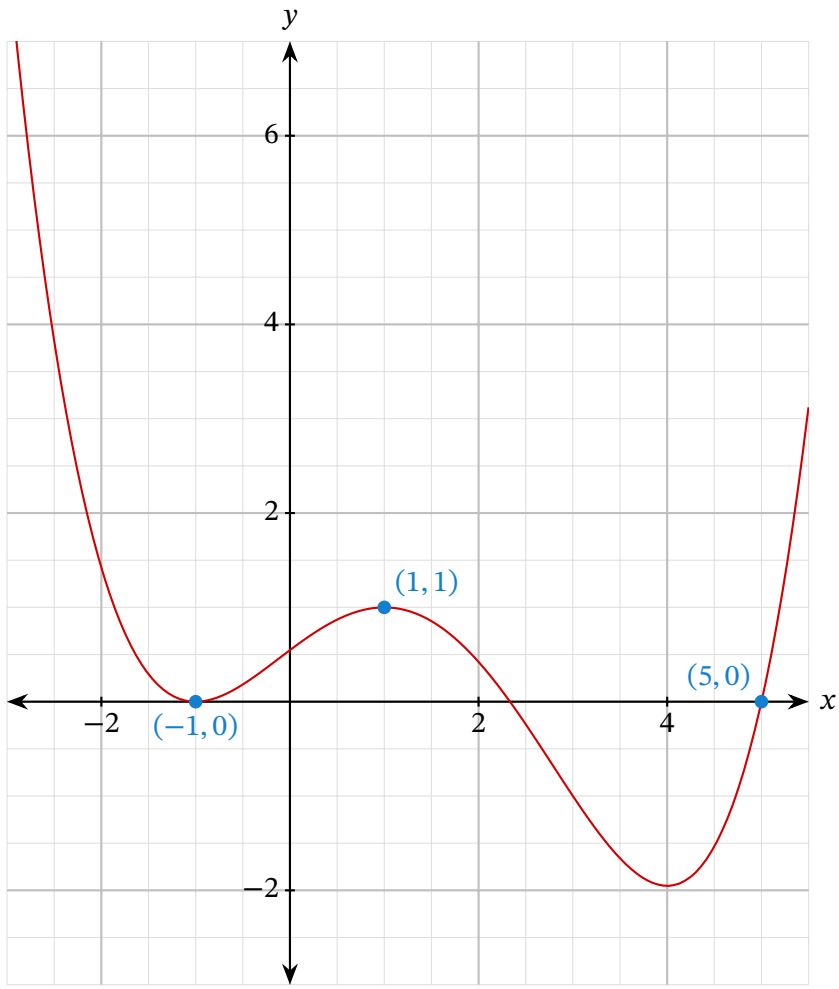
A



B



Q2: Consider the polynomial function whose graph $y = P(x)$ is given below.



Use the given points and the fact that $x = 1$ is a critical point of the function P to determine $P(x)$.

A $\frac{1}{64}(x-5)(x+1)(3x-7)$

B $\frac{1}{64}(x-5)(x+1)(3x-7)^2$

C $\frac{1}{64}(x-5)^2(x+1)(3x-7)$

D $\frac{1}{64}(x-5)(x+1)^2(3x-7)$

E $(x-5)(x+1)^2(3x-7)$

Determine the intervals where $P(x) < 1$.

A $\left(-\frac{4\sqrt{7}-5}{3}, \frac{4\sqrt{7}+5}{3}\right)$

B $\left(-\frac{4\sqrt{7}-5}{3}, 1\right) \cup (1, \infty)$

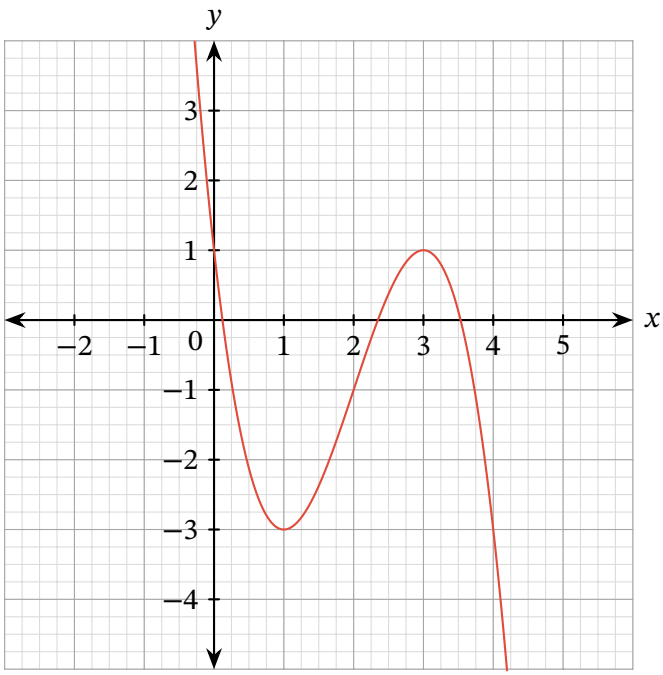
C $\left(-\infty, -\frac{4\sqrt{7}-5}{3}\right) \cup \left(1, \frac{4\sqrt{7}+5}{3}\right)$

D $\left(-\frac{4\sqrt{7}-5}{3}, 1\right) \cup \left(1, \frac{4\sqrt{7}+5}{3}\right)$

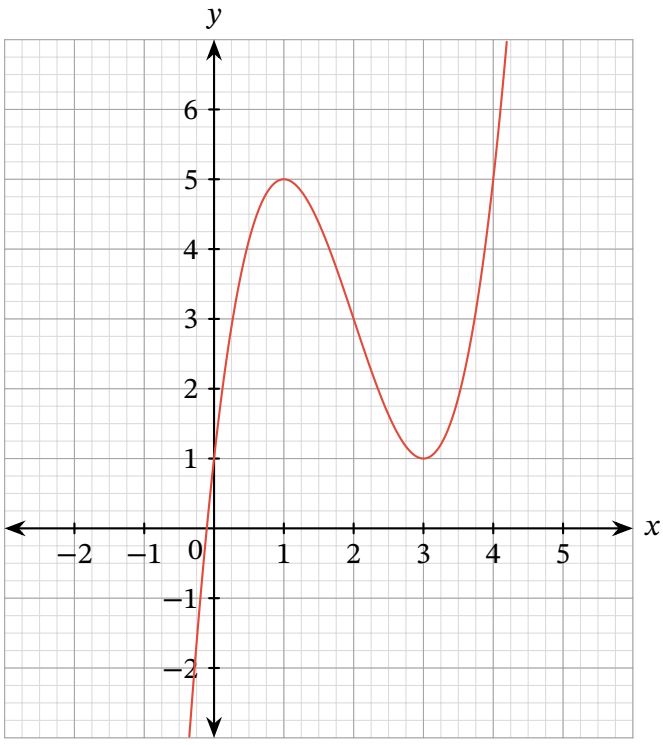
E $\left(-\infty, -\frac{4\sqrt{7}-5}{3}\right) \cup \left(\frac{4\sqrt{7}+5}{3}, \infty\right)$

Q3: Which of the following is the graph of the function $f(x) = -x^3 + 6x^2 - 9x + 1$?

A



B



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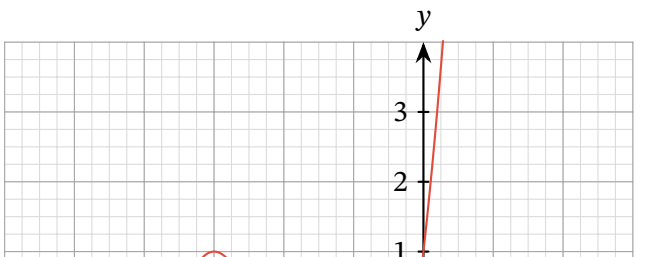
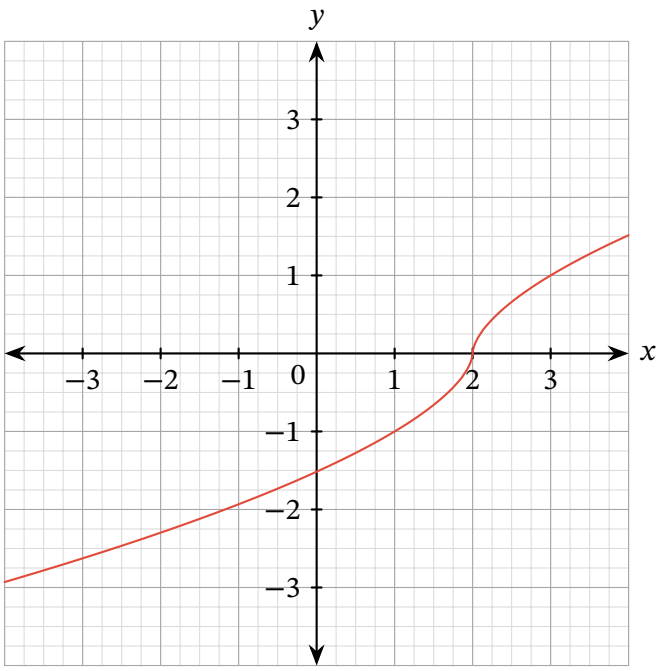
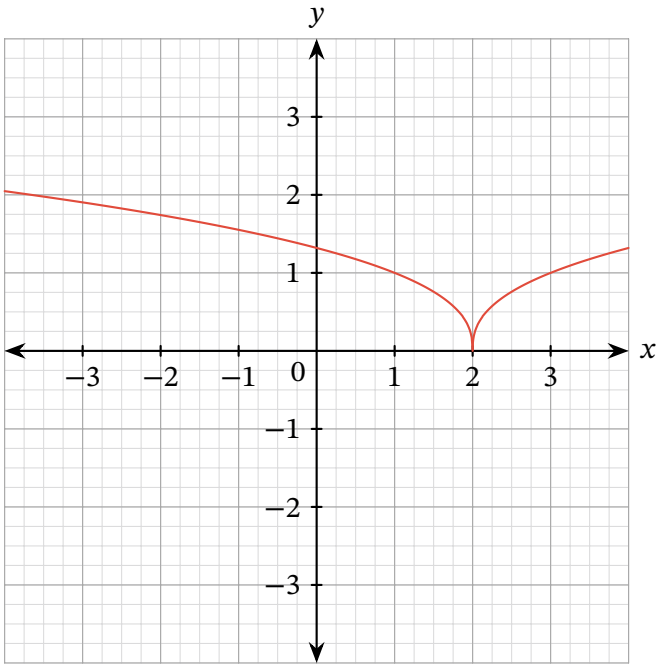


Q4: Which of the following could be the graph of the function $f(x) = (x - 2)^{\frac{3}{5}}$? Use derivatives to sketch the function.

A



B



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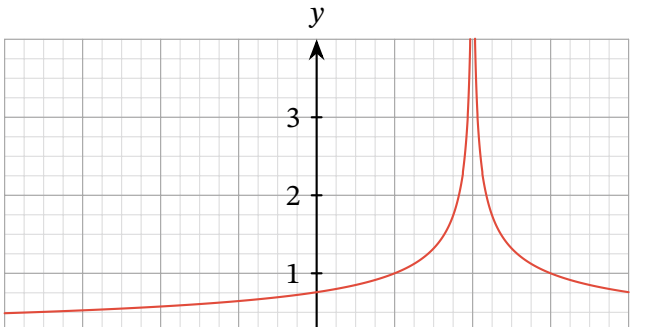
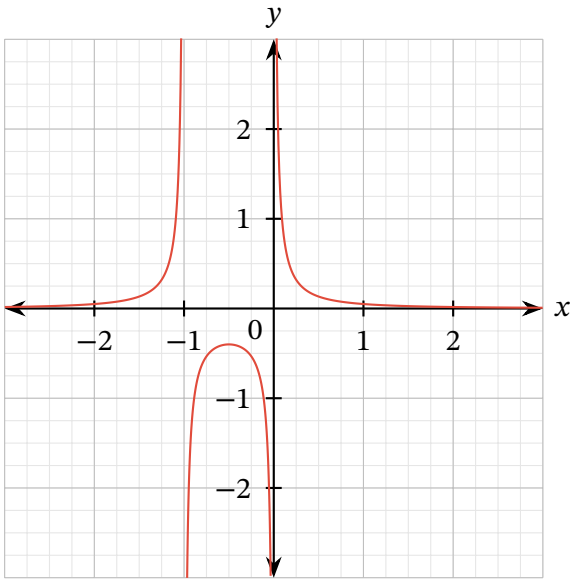


Q5: Which of the following is the graph of the function $f(x) = \dfrac{1}{10x^2 + 10x}$?

A



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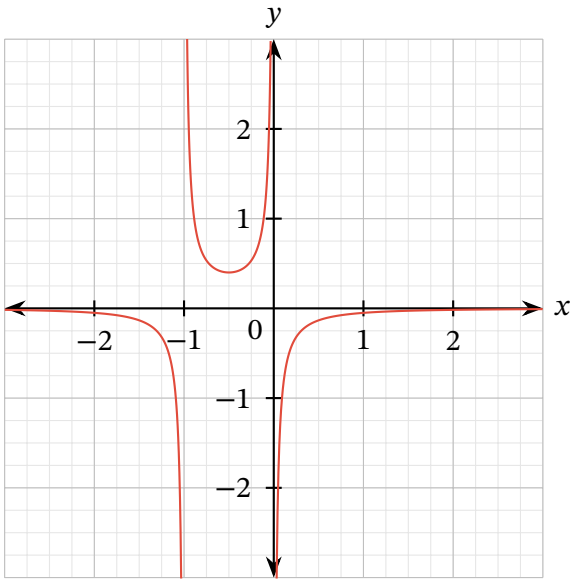
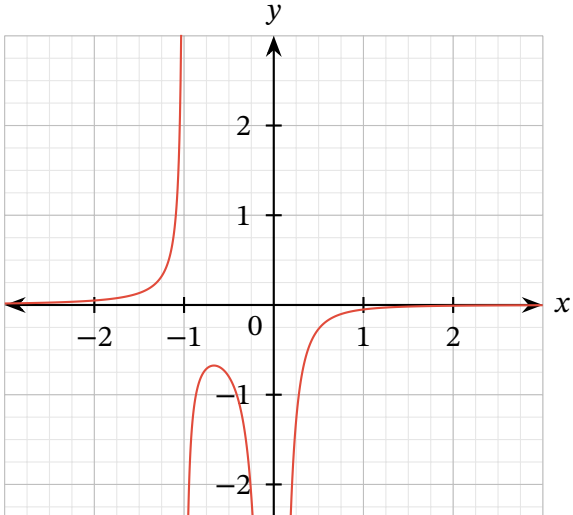
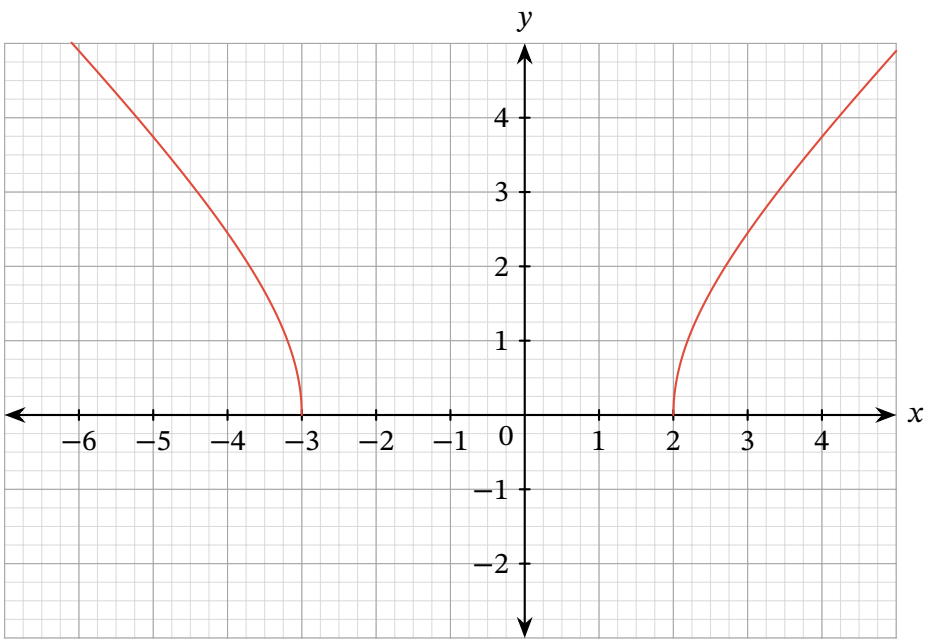


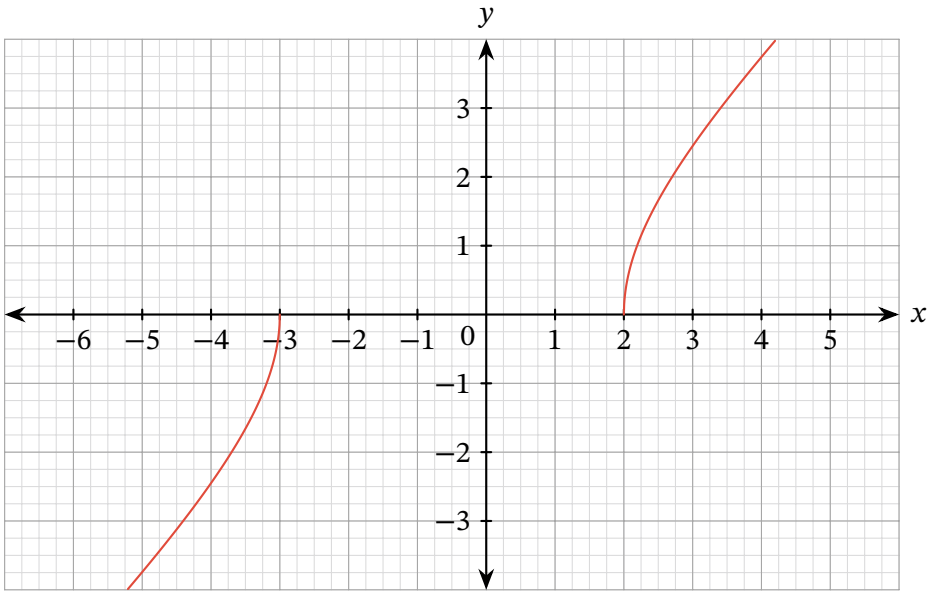


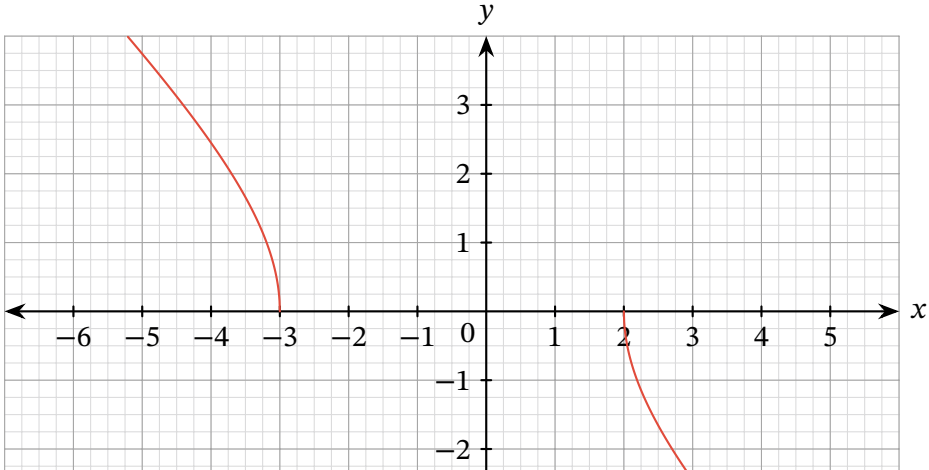
Q6: Which of the following is the graph of the function $f(x) = \sqrt{x^2 + x - 6}$?

A

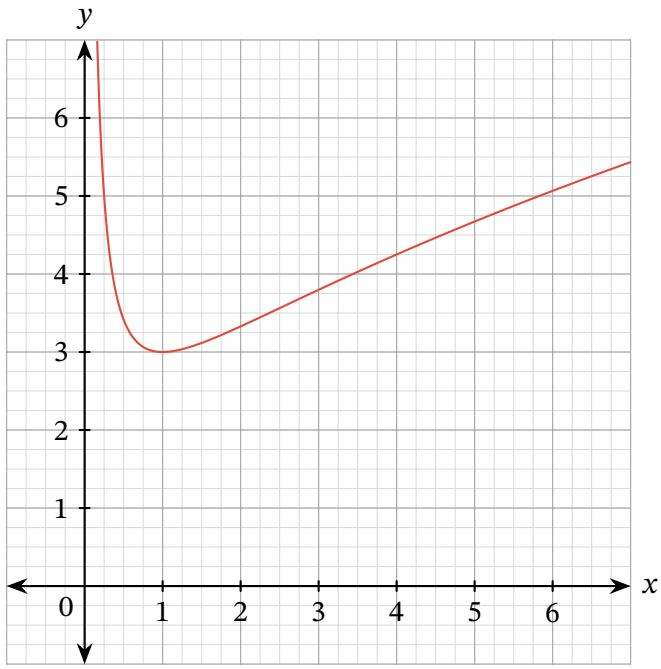


B

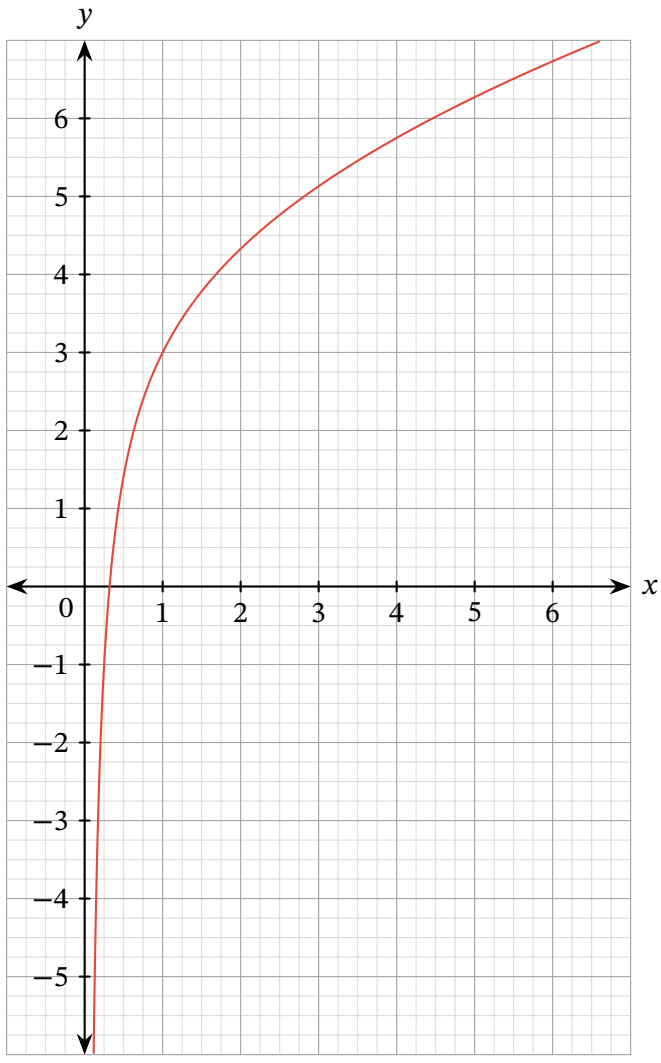




Q7: Which of the following is the graph of the function $f(x) = \frac{1}{x} + 2\sqrt{x}$?

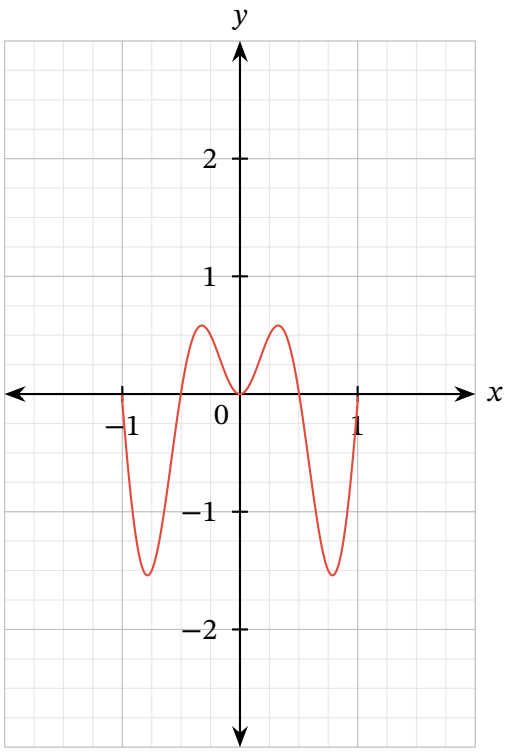


A

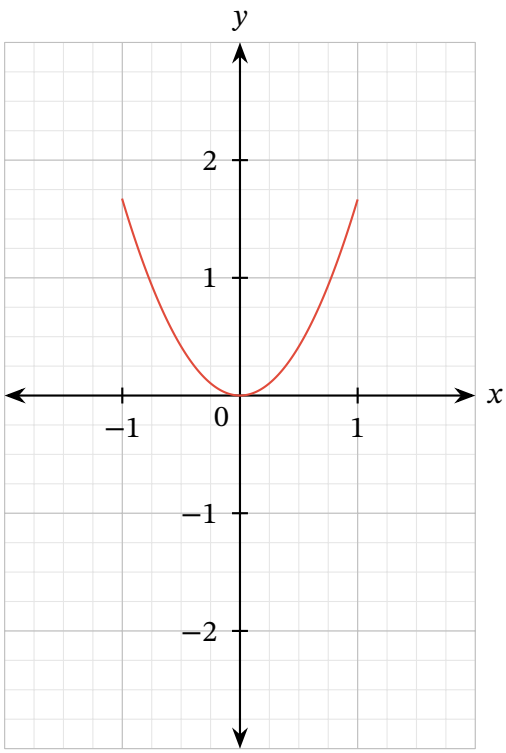


B

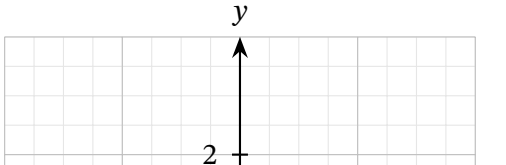
Q8: Which of the following is the graph of the function $f(x) = 2x \sin 2\pi x$ on the interval $[-1, 1]$?



A



B



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Q9: Consider the function $f(x) = \dfrac{x^2 - x + 2}{5x - 10}$.

- Find all asymptotes of f.

A f has a vertical asymptote at $y = 2$ and an oblique asymptote at $y = \frac{1}{5}x - \frac{3}{5}x$.

B f has a vertical asymptote at $x = 2$ and an oblique asymptote at $y = \frac{1}{5}x + \frac{1}{5}$.

C f has a horizontal asymptote at $y = 2$ and an oblique asymptote at $y = \frac{1}{5}x + \frac{1}{5}$.

D f has a vertical asymptote at $x = 2$ and an oblique asymptote at $y = \frac{1}{5}x - \frac{3}{5}x$.

E f has a horizontal asymptote at $x = 2$ and an oblique asymptote at $y = \frac{1}{5}x$.

- Find $f'(x)$.

A $\dfrac{-10x^2 + 35x - 15}{(5x - 10)^2}$

B $\dfrac{5x^2 - 20x}{(5x - 10)^2}$

C $\dfrac{20x - 5x^2}{(5x - 10)^2}$

D $\dfrac{2x - 1}{5}$

E $\dfrac{10x^2 - 35x + 15}{(5x - 10)^2}$

Find and classify all critical points of f.

A f has neither a local maximum nor a local minimum at $x = 2$.

B f has a local maximum at $x = 0$ and a local minimum at $x = 4$.

C f has a local minimum at $x = \frac{1}{2}$ and a local maximum at $x = 3$.

D f has a local minimum at $x = 3$ and a local maximum at $x = 4$.

E f has a local maximum at $x = \frac{1}{2}$ and a local minimum at $x = 3$.

Find the intervals of increase and decrease for f.

A f is increasing on $(-\infty, -4)$ and $(0, \infty)$ and decreasing on $(-4, 0)$.

B f is increasing on $(-\infty, 0)$ and $(4, \infty)$ and decreasing on $(0, 2)$ and $(2, 4)$.

C f is increasing on $\left(-\infty, \frac{1}{2}\right)$ and $(3, \infty)$ and decreasing on $\left(\frac{1}{2}, 2\right)$ and $(2, 3)$.

D f is increasing on $(0, 2)$ and $(2, 4)$ and decreasing on $(-\infty, 0)$ and $(4, \infty)$.

E f is increasing on $\left(\frac{1}{2}, 2\right)$ and $(2, 3)$ and decreasing on $\left(-\infty, \frac{1}{2}\right)$ and $(3, \infty)$.

Find $\lim_{x \to \infty} f(x)$.

A $-\frac{1}{5}$

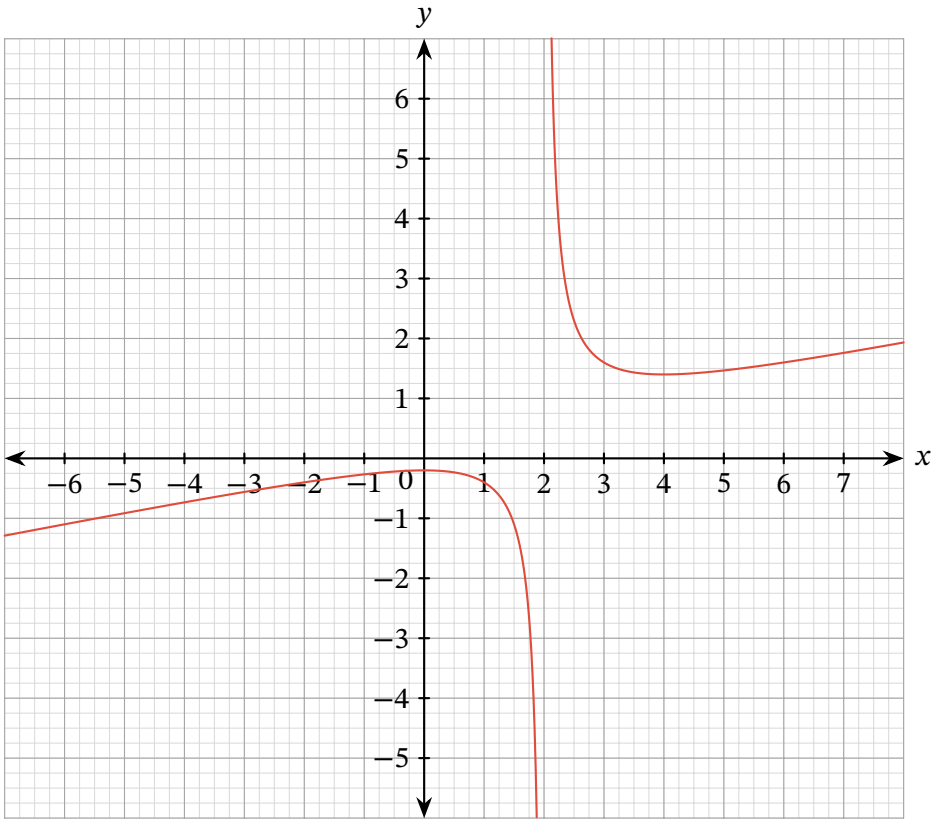
B ∞

C $\frac{1}{5}$

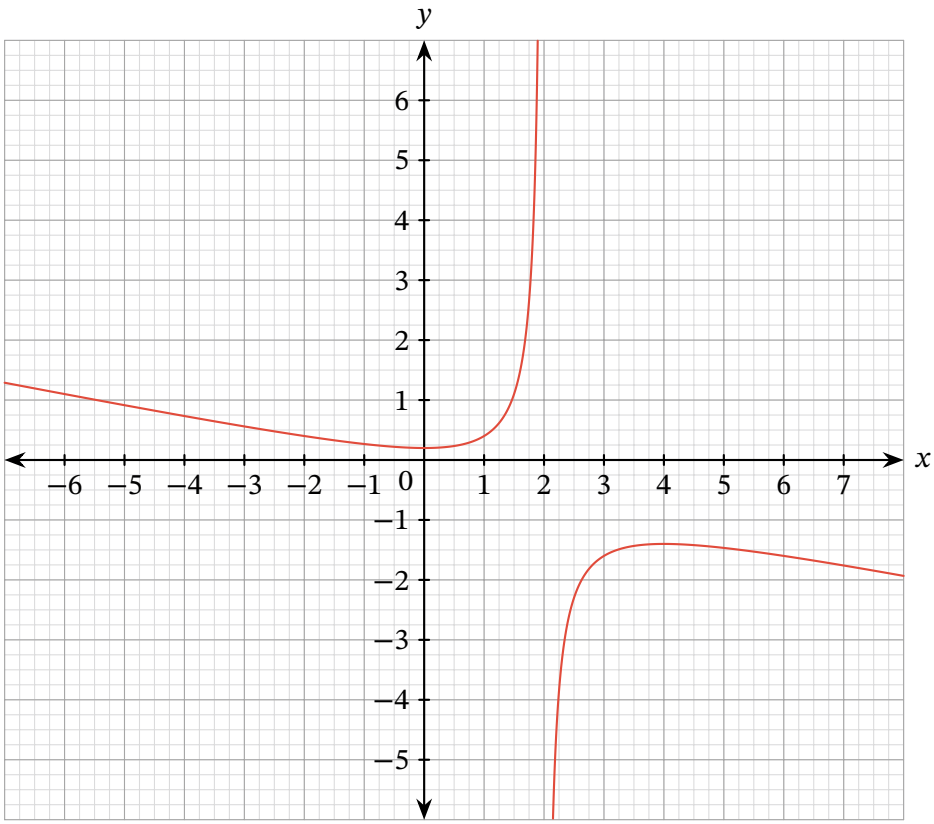
D 0

E $-\infty$

Which of the following could be the graph of f?



A



B

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Q10: Consider the function $f(x) = (x-1)^2(x+2)$.

- Find $f'(x)$.

A $2x+1$

B $3x^2-3$

C $2x-2$

D $4x+1$

E x^2+2x

- Find and classify the critical points of f.

A f has a local minimum at $x = -\frac{1}{2}$.

B f has a local minimum at $x = 1$ and a local maximum at $x = -1$.

C f has a local maximum at $x = -\frac{1}{2}$.

D f has a local minimum at $x = 0$ and a local maximum at $x = -2$.

E f has a local maximum at $x = 1$ and a local minimum at $x = -1$.

Find the intervals of increase and decrease for f.

A f is increasing on $\left(-\infty, -\frac{1}{2}\right)$ and decreasing on $\left(-\frac{1}{2}, \infty\right)$.

B f is increasing on $(-\infty, -1)$ and $(1, \infty)$ and decreasing on $(-1, 1)$.

C f is increasing on $(-1, 1)$ and decreasing on $(-\infty, -1)$ and $(1, \infty)$.

D f is increasing on $(-\infty, -2)$ and $(0, \infty)$ and decreasing on $(-2, 0)$.

E f is increasing on $\left(-\frac{1}{2}, \infty\right)$ and decreasing on $\left(-\infty, -\frac{1}{2}\right)$.

Find $\lim_{x \to \infty} f(x)$.

A 0

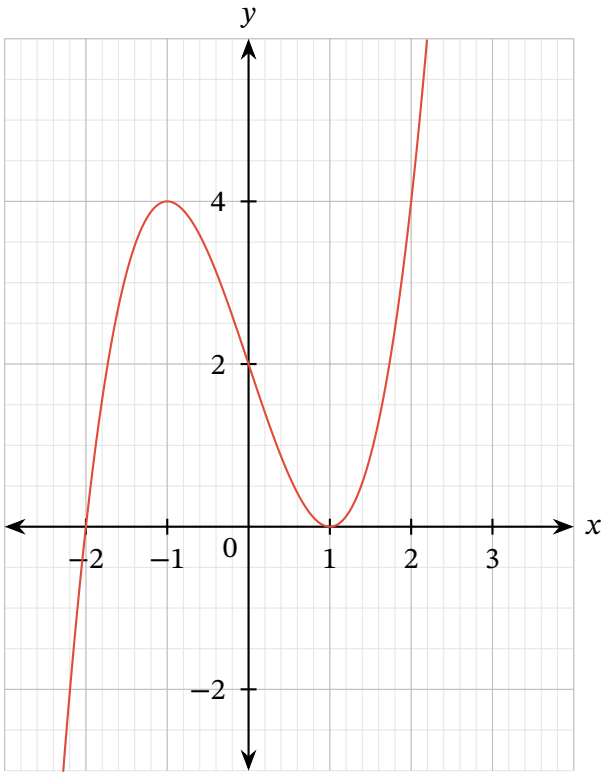
B ∞

C $-\infty$

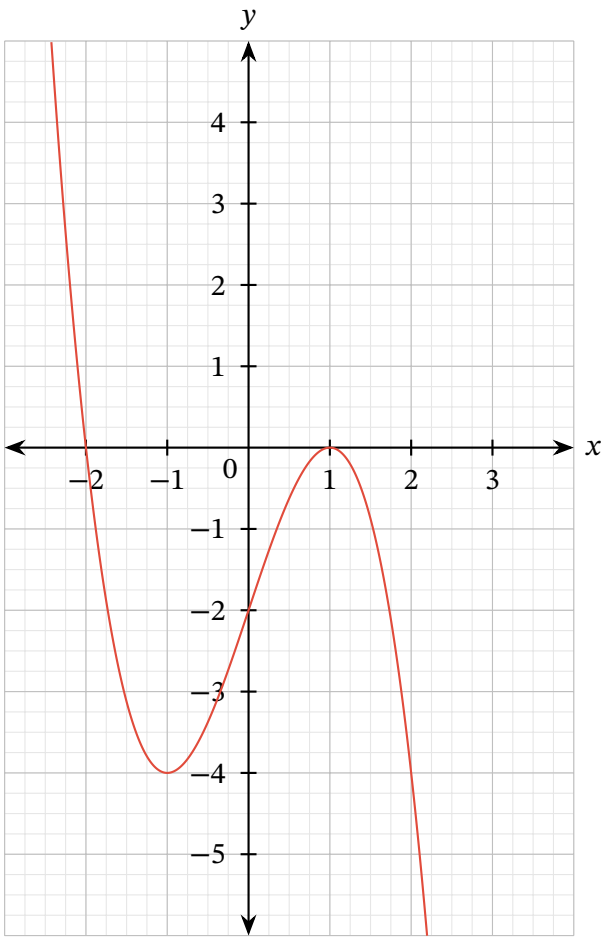
D -2

E 1

Which of the following is the graph of f?



A



B

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