

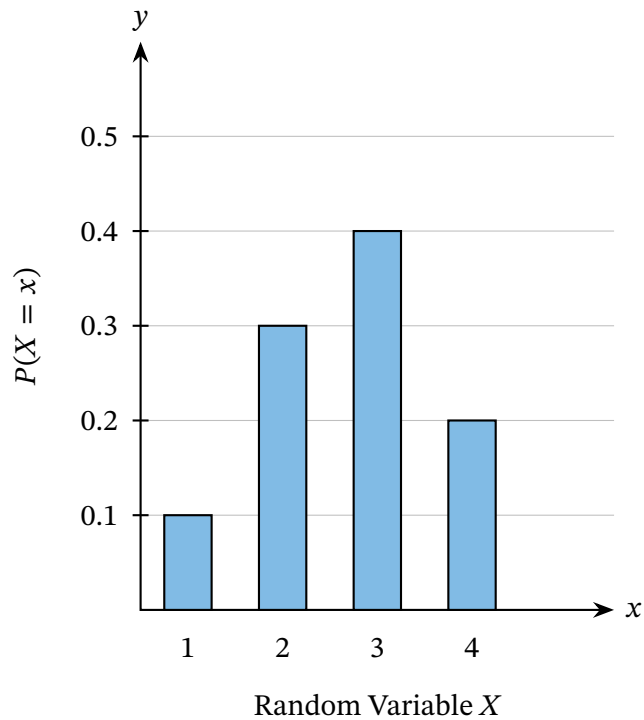
# Worksheet: Expected Values of Discrete Random Variables



**Q1:** Work out the expected value of the random variable  $X$  whose probability distribution is shown.



Question Video



A 2.5

B 2.7

C 2.4

D 3

E 4

**Q2:** The function in the given table is a probability function of a discrete random variable  $X$ . Find the value of  $a$ .



Question Video

$x_i$	0	1	2	3	4
$f(x_i)$	$2a$	0.3	0.3	$a$	$a$

- A 0
- B 0.9
- C 0.2
- D 0.1

**Q3:** The function in the given table is a probability function of a discrete random variable  $X$ . Find the expected value of  $X$ .



Question Video

$x_i$	1	3	4	6
$f(x_i)$	$\frac{10}{27}$	$8a$	$6a$	$\frac{1}{9}$

- A 15
- B  $\frac{286}{27}$
- C  $\frac{58}{27}$
- D  $\frac{76}{27}$

**Q4:** The frequency table shows the number of cars that 65 families have.



Question Video

Number of Cars	1	2	3	4
Frequency	10	35	15	5

► Find the mean number of cars per family.

A  $\frac{29}{13}$

B  $\frac{2}{29}$

C  $\frac{13}{2}$

D  $\frac{29}{2}$

E  $\frac{13}{29}$

► This data can be expressed as a probability distribution for the discrete random variable  $X$  as shown. Find the value of  $a$ ,  $b$ ,  $c$ , and  $d$ .

$x$	1	2	3	4
$P(x)$	$a$	$b$	$c$	$d$

- A  $a = \frac{2}{13}, b = \frac{7}{13}, c = \frac{3}{13}, d = \frac{1}{13}$
- B  $a = \frac{11}{13}, b = \frac{6}{13}, c = \frac{10}{13}, d = \frac{12}{13}$
- C  $a = \frac{2}{13}, b = \frac{7}{13}, c = \frac{9}{13}, d = \frac{4}{13}$
- D  $a = \frac{1}{10}, b = \frac{2}{35}, c = \frac{1}{5}, d = \frac{4}{5}$
- E  $a = \frac{2}{13}, b = \frac{7}{13}, c = \frac{9}{13}, d = \frac{1}{13}$

► Calculate the expected value of  $X$ .

A  $\frac{29}{13}$

B  $\frac{2}{29}$

C  $\frac{13}{2}$

D  $\frac{29}{2}$

E  $\frac{13}{29}$

**Q5:** The table shows the probability distribution of a fair six-sided die. Determine  $E(X)$ .



Question Video

$x$	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

A 2.3

B 1

C 3.5

D 3.3

E 6

**Q6:** The discrete random variable  $X$  has the shown probability distribution.



Question Video

$x$	1	2	3	4	5	6
$p(x)$	0.1	0.3	0.2	0.1	0.1	$k$

► Find the value of  $k$ .

- A 0.4
- B 0.1
- C 0.8
- D 0.2
- E 0.3

► Hence, determine the expected value of  $X$ .

- A 4.6
- B 2.8
- C 7
- D 3.4
- E 4

**Q7:** An experiment produces the discrete random variable  $X$  that has the probability distribution shown. If a very high number of trials were carried out, what would be the likely mean of all the outcomes?



Question Video

$x$	2	3	4	5
$p(x)$	0.1	0.3	0.2	0.4

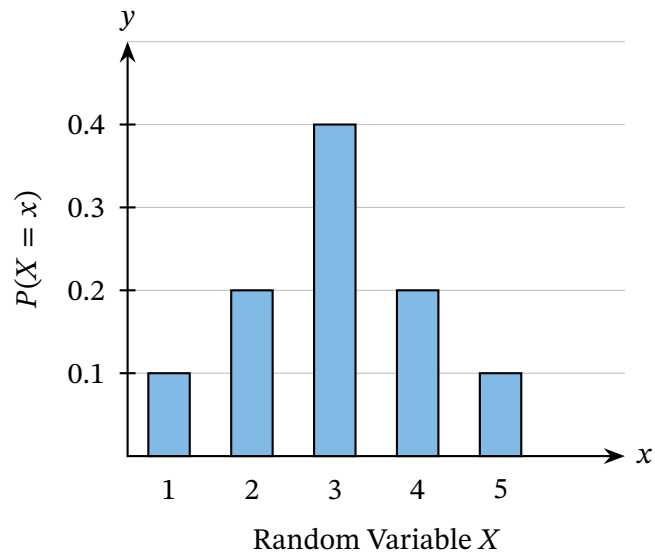
- A 4
- B 3
- C 3.9
- D 3.7
- E 1.9



**Q8:** Work out the expected value of the random variable  $X$  whose probability distribution is shown.



Question Video

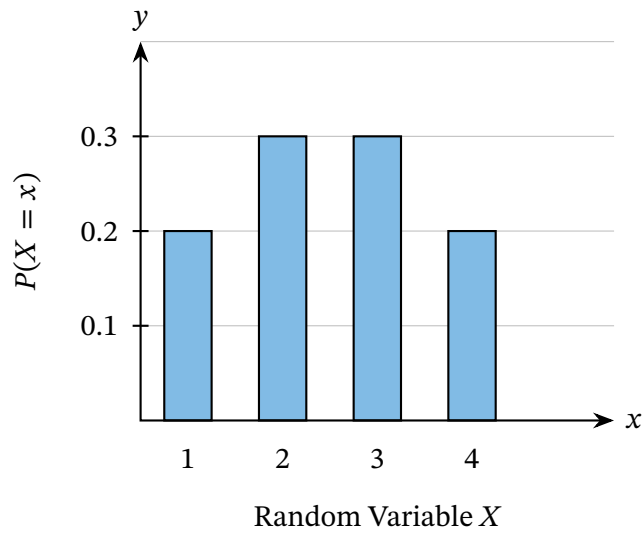


- A 3.6
- B 2
- C 4.2
- D 3
- E 2.7

**Q9:** Work out the expected value of the random variable  $X$  whose probability distribution is shown.



Question Video



- A 3
- B 2
- C 2.5
- D 1.5
- E 2.1

**Q10:** The function in the given table is a probability function of a discrete random variable  $X$ . Given that the expected value of  $X$  is 4, find the values of  $a$  and  $b$ .



Question Video

$x_i$	1	3	$b$	5	6
$f(x_i)$	0.2	0.2	$a$	0.2	0.3

- A  $a = 0.2, b = 5$
- B  $a = 0, b = 3$
- C  $a = 0.1, b = 4$
- D  $a = 0.1, b = 3$

**Q11:** The function in the given table is a probability function of a discrete random variable  $X$ . Find the value of  $a$ .



Question Video

$x_i$	2	3	4	5
$f(x_i)$	$7a$	$5a$	$9a$	$3a$

- A  $\frac{1}{7}$
- B 0
- C  $\frac{1}{12}$
- D  $\frac{1}{24}$

**Q12:** An experiment that produces the discrete random variable  $X$  has the probability distribution shown.



Question Video

$x$	2	3	4	5
$p(x)$	0.1	0.3	0.2	0.4

► Calculate  $E(X)$ .

A 1.9

B 4

C 3

D 3.7

E 3.9

► Calculate  $E(X^2)$ .

A 1.25

B 15.21

C 10

D 13.7

E 16.3

► The variance of  $X$  can be calculated using the formula  $\text{Var}(X) = E(X^2) - E(X)^2$ . Calculate  $\text{Var}(X)$  to 2 decimal places.

A 0.99

B 0.30

C 0.01

D 0.65

E 1.09

**Q13:** The function in the given table is a probability function of a discrete random variable  $X$ . Given that the expected value of  $X$  is  $\frac{254}{57}$ , find the value of  $B$ .



Question Video

$x_i$	1	2	$B$	7
$f(x_i)$	$8a$	$3a$	$\frac{1}{3}$	$8a$

A 4

B 5

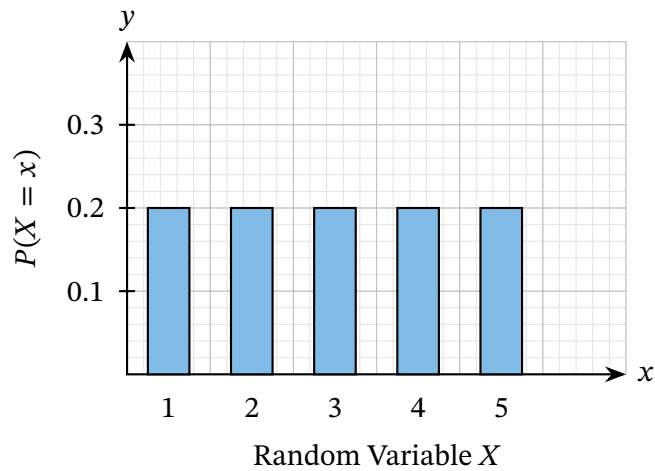
C 7

D 6

**Q14:** Work out the expected value of the random variable  $X$  whose probability distribution is shown.



Question Video



A 11

B 2

C 1.5

D 4.5

E 3

**Q15:** Scarlett had a spinner with ten equal sections labeled with the numbers 1 to 10. She spun it 300 times and recorded the outcomes in a frequency table.



Question Video

Number	1	2	3	4	5	6	7	8	9	10
Frequency	35	27	22	11	24	28	33	35	49	36

► If the spinner was fair, how many times would you expect to see each number if you spun it 300 times?

A 30

B 10

C 15

D 40

E 20

► State whether the spinner is biased and why.

A The spinner is biased because the number 4 only appeared half as often as expected and the number 9 appeared much more often than expected.

B The spinner is not biased because most of the numbers appeared around 30 times.

C The spinner is biased because the numbers did not appear exactly 30 times each.



**Q16:** The function in the given table is a probability function of a discrete random variable  $X$ . Find the expected value of  $X$ .



Question Video

$x_i$	0	1	2	3	4
$f(x_i)$	0.1	$a$	0.1	0.4	0.2

- A 7.4
- B 1.6
- C 2.4
- D 11

**Q17:** Let  $X$  denote a discrete random variable which can take the values  $-1, M$ , and  $1$ . Given that  $X$  has probability distribution function  $f(x) = \frac{x+2}{6}$ , find the expected value of  $X$ .



Question Video

- A  $\frac{8}{3}$
- B  $\frac{1}{3}$
- C 1
- D  $\frac{2}{3}$

**Q18:** Let  $X$  denote a discrete random variable which can take the values 1, 2, 3, 4, and 5. Given that  $P(X = 1) = \frac{7}{33}$ ,  $P(X = 2) = \frac{8}{33}$ ,  $P(X = 3) = \frac{1}{11}$ , and  $P(X = 4) = \frac{1}{33}$ , find the expected value of  $X$ .



Question Video

- A 16
- B  $\frac{106}{33}$
- C  $\frac{144}{11}$
- D  $\frac{12}{11}$

**Q19:** Let  $X$  denote a discrete random variable which can take the values 4, 5, 8, and 10. Given that  $P(X = 4) = \frac{4}{27}$ ,  $P(X = 5) = \frac{5}{27}$ , and  $P(X = 8) = \frac{8}{27}$ , find the expected value of  $X$ . Give your answer to two decimal places.



Question Video

- A 0.72
- B 8.59
- C 28.00
- D 7.59

**Q20:** Let  $X$  denote a discrete random variable which can take the values  $-2, 0,$  and  $5$ . Given that the expectation of  $X$  is  $0.03$  and  $P(X = -2) = \frac{9}{25}$ , find  $P(X = 5)$ .



Question Video

A  $\frac{16}{25}$

B  $\frac{51}{100}$

C  $\frac{3}{20}$

D  $\frac{9}{25}$

**Q21:** The discrete random variable  $X$  has the shown probability distribution.



Question Video

$x$	1	2	3	4
$P(x)$	$\frac{k}{1}$	$\frac{k}{2}$	$\frac{k}{3}$	$\frac{k}{4}$

► Find the value of  $k$ .

A  $\frac{12}{25}$

B  $\frac{12}{13}$

C  $\frac{1}{10}$

D  $\frac{6}{11}$

E  $\frac{25}{12}$

► Hence, determine the expected value of  $X$ .

A  $\frac{48}{25}$

B  $\frac{48}{13}$

C  $\frac{2}{5}$

D  $\frac{24}{11}$

E  $\frac{25}{3}$

**Q22:** A discrete random variable  $X$  has a uniform probability distribution such that  $P(X = x) = \frac{1}{11}$ , where  $x \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ . Determine  $E(X)$ .



Question Video

A 1

B 6

C 5

D 4

E 7

**Q23:** 23 students took an exam; 7 students got 3 marks, 8 students got 8 marks, and 8 students got 2 marks. Given that  $X$  denotes the number of marks received, find the expected value of  $X$ . If necessary, round your answer to the nearest hundredth.



Question Video

A 0.34

B 2.00

C 14.00

D 4.39

**Q24:** In an experiment, Emma is going to spin a fair four-sided spinner numbered from 1 to 4. Chloe says that the expected value of the experiment is 2.5. Emma disagrees as she says it is impossible to spin 2.5 and suggests that the expected value is 3. Who is correct and why?



Question Video

- A** Emma is correct because the expected value is the average result of an experiment after a large number of trials, which is 2.5 in this case. However, this is unobtainable on the spinner, so it must be rounded to the nearest whole number, which is 3.
- B** Chloe is correct because the expected value is the average result of an experiment after a large number of trials, which is 2.5 in this case.

**Q25:** In an experiment, Hannah rolls two fair six-sided dice and adds the numbers. The probability distribution of the experiment is shown.

$x$	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$a$	$\frac{4}{36}$	$b$	$c$	$\frac{5}{36}$	$d$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

► Find the value of  $a$ ,  $b$ ,  $c$ , and  $d$ .

- A**  $a = \frac{3}{36}, b = \frac{5}{36}, c = \frac{6}{36}, d = \frac{4}{36}$
- B**  $a = \frac{3}{36}, b = \frac{5}{36}, c = \frac{1}{36}, d = \frac{4}{36}$
- C**  $a = \frac{3}{36}, b = \frac{5}{36}, c = \frac{6}{36}, d = \frac{7}{36}$
- D**  $a = \frac{3}{36}, b = \frac{5}{36}, c = \frac{1}{36}, d = \frac{3}{36}$
- E**  $a = \frac{5}{36}, b = \frac{3}{36}, c = \frac{6}{36}, d = \frac{4}{36}$

► What is the expected value of the experiment?

A 7

B 6

C 7.75

D 5.7

E 6.75