Explainer: Writing the Equation of a Given Circle Knowing Its Center

In this explainer, we will learn how to find the equation of a circle when given its center and radius or its center and a point it passes through.

For this topic, you should know the general equation of a circle.

Equation of a Circle in Center-Radius Form

The equation of a circle centered at $C(h, k)$ and of radius r is

$$(x - h)^2 + (y - k)^2 = r^2.$$

It should be noted that, by expanding the brackets and rearranging, the previous equation is found to be equivalent to

$$x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0.$$

In this way, the general form of the equation of the circle can be derived.

Equation of a Circle in General Form

The equation of a circle can be written in the form

$$x^2 + y^2 + ax + by + c = 0.$$

We are going to learn how to use the information we have about a circle to write its equation.

Example 1: Writing the Equation of a Circle Given Its Center

What is the equation of the circle of radius 10 and center $(4, -7)$?

Give your answer in the form $x^2 + y^2 + ax + by + c = 0$.

Answer

We start by writing the equation of a circle:

$$(x - h)^2 + (y - k)^2 = r^2.$$

The radius r is 10 and the center coordinates are $h = 4$ and $k = -7$, so this gives us

$$(x - 4)^2 + (y + 7)^2 = 10^2$$

$$(x-4)^2+(y+7)^2=100.$$

This is the equation of the circle of radius 10 and center $(4, -7)$ in center-radius form.

However, we are asked to give it in the form $x^2+y^2+ax+by+c=0$.

We need to expand the brackets,

$$x^2-8x+16+y^2+14y+49=100,$$

and then take away 100 from each side,

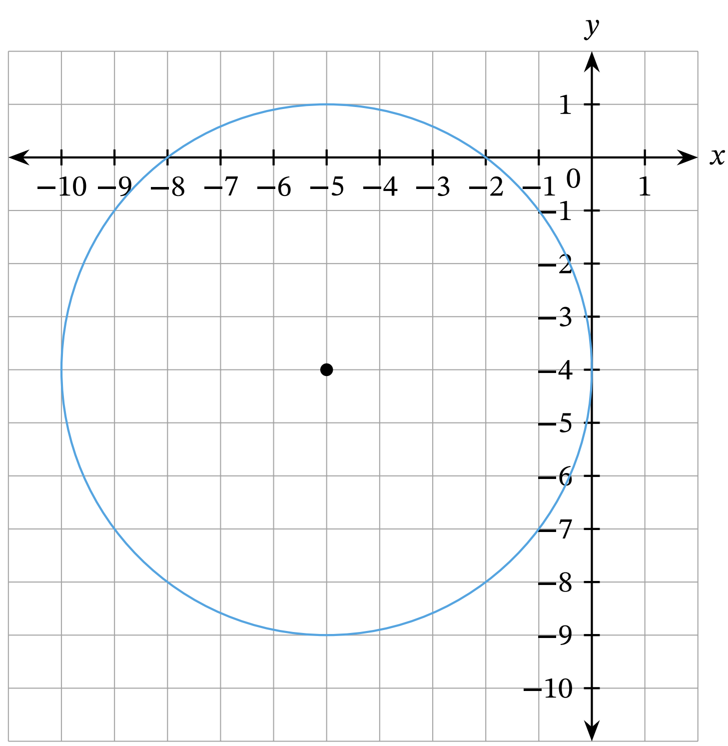
$$x^2-8x+16+y^2+14y+49-100=0,$$

and collect like terms:

$$x^2+y^2-8x+14y-35=0.$$

Example 2: Writing the Equation of a Circle Given Its Center

In the figure below, find the equation of the circle.



Answer

In this example, we need to use the graph to identify the center's coordinates and the radius of the circle.

The circle center's coordinates are $(h, k) = (-5, -4)$.

To find the radius, we can, for instance, work out the difference in the y-coordinates of the highest point and the center, $1 - (-4) = 1 + 4 = 5$, or the difference in the x-coordinates of the furthest point to the right and the center: $0 - (-5) = 5$. So $r = 5$.

We plug the values of h, k, and r in $(x-h)^2+(y-k)^2 = r^2$ and find $(x+5)^2+(y+4)^2 = 25$.

Example 3: Writing the Equation of a Circle Given Its Center

Determine the equation of a circle that passes through the point $A(0, 8)$ if its center is $M(-2, -6)$.

Answer

We start by writing the general equation of a circle:

$$(x - h)^2 + (y - k)^2 = r^2.$$

We know that point $M(-2, -6)$ is the center of the circle, so $h = -2$ and $k = -6$. We then plug in these values in the equation, and we get

$$(x + 2)^2 + (y + 6)^2 = r^2.$$

We do not know the radius, but we know that point A is on the circle, so its coordinates $x = 0$ and $y = 8$ must satisfy the equation of the circle. We can, therefore, substitute x and y in the equation with these values to find r:

$$\begin{aligned} (2)^2 + (8 + 6)^2 &= r^2 \\ 4 + 196 &= r^2 \\ 200 &= r^2. \end{aligned}$$

The equation of the circle is finally

$$(x + 2)^2 + (y + 6)^2 = 200.$$