Explainer: Expected Values

In this explainer, we will learn how to use theoretical and experimental probabilities to calculate expected values.

When we talk of the probability of something happening, we mean the chance or likelihood of it occurring. The probability of an event tells us how likely something is to happen in the long run, and there are two ways of looking at this.

1. **Experimental probability** is calculated by looking at the outcomes of a repeated experiment. Experimental probability is also sometimes called the "relative frequency."
2. **Theoretical probability** is calculated using mathematical reasoning about the possible outcomes.

Theoretically, we know that if we toss a fair coin, we have a 50% (or a 1 in 2) chance of getting "heads"; this is the theoretical probability. If "H" stands for "heads," we write $P(H) = 0.5$; that is, the probability of "heads" is 0.5.

If we were to toss a coin 100 times, however, we might find that we get "heads" on only 45 out of the 100 throws. So the experimental probability of "heads" in this case is $\frac{45}{100}$, or $P(H) = 0.45$, which is 45% as a percentage, less than our theoretical probability.

The **expected value** is what we **expect** to happen on average based on either theoretical or experimental probabilities if we were to perform many trials or experiments. The expected value is calculated by multiplying the probability of the event occurring by the number of times the experiment is performed. Let us see how the expected value works in some examples.

Example 1: Expected Values for Multiple-Choice Tests

Unfortunately for Mason, just as he is about to review for his math test, he finds that the dog has eaten all of his carefully taken notes.

The test is multiple choice, with only one possible correct answer out of 4 options for each question. Mason has a very bad memory and will have to guess the answer to each question. Theoretically, then, for each question Mason will have a 1 in 4 chance of getting it right.

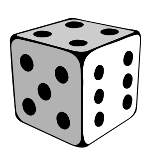
If there are 60 questions, what is Mason's expected score?

Answer

Mason's expected score is the probability of getting one question right, multiplied by the number of questions. The probability that Mason gets a single question right is $\frac{1}{4}$. Since there are 60 questions, his expected score is, therefore,

$$\begin{aligned}\text{expected score} &= \text{probability} \times \text{number of questions}\\ &= \frac{1}{4} \times 60\\ &= 15.\end{aligned}$$

Example 2: Expected Value with a Biased Die



The probability that a biased die will land on an even number is 0.6. If the die is rolled 80 times, how many times is it expected to land on an even number?

Answer

We know that the expected value is calculated by multiplying the probability of an event occurring by the number of times the experiment is performed. In this case, the event is that an even number occurs and it has a probability of 0.6. The experiment is rolling the die, which is performed 80 times.

The number of times we would expect the die to land on an even number is, therefore,

$$0.6 \times 80 = 48.$$

In our next example, we will use the experimental probability to calculate the expected value.

Example 3: Using the Experimental Probability to Calculate the Expected Value

In a survey of 400 tourists who visited Egypt, 160 were from Arab countries, 120 were from Europe, 40 were from Latin America, and 80 were from Australia. If the total number of tourists who visited Egypt in a month was 5,000, how many of them are expected to be from Europe?

Answer

To find the expected number of tourists from Europe out of 5,000, we must first work out the experimental probability that a tourist was from Europe. (The probability is classed as experimental, as opposed to theoretical, since the data come from a survey.)

There were 400 tourists surveyed, of which 120 were from Europe. Hence, the probability that a tourist chosen at random was from Europe is

$$\begin{aligned} P(\text{European}) &= \frac{\text{number of European tourists}}{\text{total number of tourists}} \\ &= \frac{120}{400} \\ &= 0.3. \end{aligned}$$

The expected value is calculated by multiplying the probability of an event occurring by the number of times the experiment is performed. In our case, we have just calculated the probability that a tourist was European as 0.3. We have a total of 5,000 tourists, so the expected number of European tourists from 5,000 is

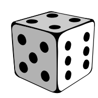
$$\text{probability} \times \text{total tourists} = 0.3 \times 5{,}000 = 1{,}500.$$

Hence, out of 5,000 tourists visiting Egypt, we would expect 1,500 of them to be from Europe.

In our next example, we will use data from a table to calculate the expected value.

Example 4: Expected Value Using Data from a Table

The table shows the results of rolling a die 78 times.

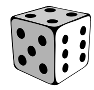


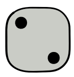
Number	Rolls
1	23
2	17
3	15
4	10
5	2
6	11

Using this information, how many times is the number 2 expected to appear if the die is rolled 234 times?

Answer

The top row in the table tells us the number on the face of the die, and the 2nd row tells us how many times a numbered face occurred out of 78 throws. This means, for example, that the number 2 occurred 17 times out of 78 throws of the die.





Number	Rolls
1	23
2	17
3	15
4	10
5	2
6	11

occured on 17 throws
out of 78

Total throws $= 23 + 17 + 15 + 10 + 2 + 11 = 78$

The probability of throwing a 2 with this die is therefore $\frac{17}{78} = 0.218$ to 3 d.p. To work out the expected number of 2s out of 234 throws, we multiply this probability (0.218) by 234. This gives us

$$\text{expected number of 2s} = 0.218 \times 234 = 51.012.$$

Of course, we cannot have 51.012 occurrences of 2, so we round this to the nearest whole number, which is 51. The expected number of 2s out of 78 throws of this die is therefore 51.

Note

We have used the experimental probability here, which we worked out from observed data: out of 78 throws of the die, 17 landed with the number 2 face up. So, the probability of throwing a 2 with this die is $\frac{17}{78} \approx 0.218$. If we had used the theoretical probability, we would have a different result.

Theoretically, each face on a die should have an equal probability of occurring. That is, the probability for each face should be the same at $\frac{1}{6}$. If we had used this probability, out of 234 throws, we would expect to get a 2, $\frac{1}{6} \times 234 = 39$ times.

Example 5: Working out Expected Values from a Data Table

A factory produces two types of shirts: A and B. A sample of 100 shirts from each of 5 shopping centers was observed to see how many of each type were sold. The results are shown in the table.

Number of shopping centers	(1)	(2)	(3)	(4)	(5)
Sales of Type A	35	66	29	44	53
Sales of Type B	65	34	71	56	47

If the factory sells 3,000 shirts, how many of them do you expect to be of type A?

Answer

To find out the expected number of shirts of type A out of 3,000 shirts sold, we must first work out the probability that a shirt sold is of type A. For this, from our table, we need the total number of type A shirts sold and the total number of shirts sold overall.

Number of shopping centers	(1)	(2)	(3)	(4)	(5)
Sales of Type A	35	66	29	44	53
Sales of Type B	65	34	71	56	47
Total:	100	100	100	100	100

Overall total shirts sold = 500

Total type A

$35 + 66 + 29 + 44 + 53 = 227$

$P\,(\text{Type A}) = \frac{227}{500} = 0.454$

The sample consists of 100 shirts from each of 5 shopping centers, so the total number of shirts overall is $5 \times 100 = 500$. The number of type A shirts sold was $35 + 66 + 29 + 44 + 53 = 227$ (35 in shopping center 1, 66 in shopping center 2, etc.). The probability that a shirt sold was of type A is, therefore,

$$P\,(\text{Type A}) = \frac{\text{number of type A shirts sold}}{\text{total number of shirts sold}}$$

$$= \frac{227}{500}$$
$$= 0.454.$$

To work out the expected number of type A shirts sold out of 3,000, we multiply our probability for a type A shirt (0.454) by 3,000:

$$\begin{aligned} \text{expected number of type A sold} &= P\,(\text{Type A}) \times 3{,}000 \\ &= 0.454 \times 3{,}000 \\ &= 1{,}362. \end{aligned}$$

Hence, we would expect 1,362 shirts out of 3,000 sold to be type A.

Let us complete our study of expected values by reminding ourselves of the main points to note.

Key Points

- The **expected value** is what we expect to happen on average over many trials or experiments. It is calculated by multiplying the probability of the event occurring by the number of times the experiment is performed:

  expected value = probability of event occurring × number of experiments or trials.

- We can often calculate the probability from data. In such cases, to find the expected value, we take the experimental probability and multiply it by the total number of trials.